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Mathematics. — *Contribution à la théorie additive des nombres.* (Cinquième Communication [1]). Par J. G. VAN DER CORPUT.

(Communicated at the meeting of December 17, 1938.)

Troisième application.

Introduisons un polynome

$$\psi(x) = b\,x^g + \ldots \qquad (b > 0, g > 0)$$

du $g^{ième}$ degré, qui prend des valeurs entières pour toutes les valeurs entières de x; introduisons en outre trois nombres naturels K, U, U', un entier u premier avec U et un entier u' premier avec U' et posons la question quels nombres naturels t possèdent la forme

$$t = Kp + \psi(p'), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

où p est un nombre premier $\equiv u \pmod{U}$ et p' un nombre premier $\equiv u' \pmod{U'}$.

Désignons par G le produit des nombres premiers p qui ne sont pas un facteur de U' et pour lesquels la congruence

$$\psi(x) \equiv \psi(1) \qquad (mod.\ p) \quad . \quad , \quad . \quad . \quad . \quad . \quad (59)$$

est vérifiée par chaque entier qui n'est pas divisible par p. Cette congruence possède alors $p-1$ ou p solutions différentes. On a donc $p \leqq g+1$ ou p est un facteur de tous les coefficients du polynome $g!\psi(x) - g!\psi(1)$. Par conséquent G est le produit d'un nombre fini de nombres premiers.

Soit E l'ensemble des nombres naturels t qui satisfont pour tout facteur premier P de $KUU'G$ la condition suivante, dans laquelle P^ω désigne la puissance la plus élevée de P qui est un diviseur de KU.

1. Si P n'est pas un facteur de U, il existe au moins un entier $X \equiv u' \pmod{U'}$ qui n'est pas divisible par P, tel que $\psi(X) - t$ soit divisible par P^ω, mais non par $P^{\omega+1}$.

2. Si P est un facteur de U, il existe au moins un entier $X \equiv u' \pmod{U'}$ qui n'est pas divisible par P, tel que $\psi(X) + Ku - t$ soit divisible par P^ω.

Considérons d'abord un nombre naturel t qui n'appartient pas à E et qui possède la susdite forme (58). Puisque t n'appartient pas à E, au moins un facteur premier P de $KUU'G$ jouit de la propriété suivante:

[1]) Voir Proc. Kon. Ned. Akad. v. Wetenschappen, A'dam, **41**, 227—237; 350—361; 442—453; 556—567 (1938).

1. Si P n'est pas un facteur de U, il n'existe aucun entier $X \equiv u'$ (mod. U') qui n'est pas divisible par P tel que $\psi(X) - t$ soit divisible par P^{ω} et non par $P^{\omega+1}$. Parce que P n'est pas un facteur de U, le nombre P^{ω} est un diviseur de K, par conséquent aussi de $-Kp \equiv \psi(p') - t$. Si p' n'est pas égal à P, la puissance $P^{\omega+1}$ est donc un diviseur de Kp, de sorte que p est divisible par, c'est à dire égal à P.

2. Si P est un facteur de U, tout entier $X \equiv u'$ (mod. U') tel que $\psi(X) + Ku - t$ soit divisible par P^{ω}, est divisible par P. Puisque $\psi(p') + Ku - t \equiv K(u - p)$ est divisible par KU, c'est-à-dire par P^{ω}, le nombre premier p' est divisible par, c'est-à-dire égal à P.

Ainsi nous avons obtenu le résultat suivant:

Si un nombre naturel t n'appartenant pas à E peut être mis sous la forme (58), où p est un nombre premier $\equiv u$ (mod. U) et p' un nombre premier $\equiv u'$ (mod. U'), au moins un de ces deux nombres premiers est un facteur de $KUU'G$.

Pour les nombres t appartenant à E je déduirai le résultat suivant:

Proposition 10: *Presque tout entier appartenant à E peut être mis sous la susdite forme (58). A tout nombre naturel m correspond un nombre c_{87}, dépendant uniquement de m, K, U, U' et du choix du polynome $\psi(x)$ tel que le nombre des exceptions $\leqq N$ soit pour tout $N \geqq 3$ inférieur à $c_{87} N (\log N)^{-m}$.*

Si nous voulons savoir quels nombres naturels t sont égaux à un nombre premier augmenté du carré d'un nombre premier, nous posons

$$K = U = U' = 1; \qquad \psi(x) = x^2;$$

G est le produit des nombres premiers p tels que tout entier X non divisible par p satisfait la congruence $X^2 \equiv 1$ (mod. p), c'est-à-dire la congruence $X \equiv \pm 1$ (mod. p), d'où il suit $G = 2.3 = 6$. L'ensemble E est formé par les nombres naturels t satisfaisant la condition suivante: à tout facteur premier p de 6 correspond au moins un entier x non divisible par p, tel que $x^2 - t$ ne soit pas divisible par p. Le facteur $p = 2$ fournit la condition que t soit pair. Le facteur $p = 3$ donne la condition que t soit congru à 0 ou 2 (mod. 3). E est donc l'ensemble des nombres naturels $\equiv 0$ ou $\equiv 2$ (mod. 6). Ainsi on obtient le résultat, mentionné au commencement de la première communication:

presque tout nombre naturel qui est congru à 0 ou 2 (mod. 6) est égal à un nombre premier, augmenté du carré d'un nombre premier.

Si nous posons

$$K = U = U' = 1, \quad \psi(x) = x^g \qquad (g \text{ entier} > 0),$$

G est le produit des nombres premiers p pour lesquels chaque entier x, non divisible par p, satisfait la congruence

$$x^g \equiv 1 \qquad (\text{mod. } p). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

Chaque nombre premier p, tel que $p - 1$ soit un diviseur de g, est donc

un facteur de G. D'autre part, si le plus grand commun diviseur d de $p-1$ et g est inférieur à $p-1$, il existe au moins un entier x, non divisible par p, tel que x^d ne soit pas congru à 1 (mod. p), d'où il suit que x ne satisfait pas à la congruence (60). G est donc le produit des nombres premiers p, tels que $p-1$ soit un diviseur de g. L'ensemble E est formé par les nombres naturels t tels qu'à tout facteur premier P de G corresponde au moins un entier X, non divisible par P, avec la propriété que $X^g - t$ ne soit pas divisible par P. En vertu de $X^g \equiv 1$ (mod. P), E est donc formé par les nombres naturels t tels que $t-1$ soit premier avec G. Ainsi nous trouvons un autre résultat, figurant dans le commencement de la première communication:
presque tout nombre naturel t tel que $t-1$ soit premier avec G, est égal à un nombre premier, augmenté de la $g^{ième}$ puissance d'un nombre premier.

On trouve le troisième cas particulier, traité dans le commencement de la première communication, en posant $K = U = U' = 1$, tandis que $\psi(x)$ désigne un polynome réel quelconque, qui prend des valeurs entières pour toutes les valeurs entières de x et qui croît indéfiniment avec x. Dans ce cas je démontrerai que E est formé par les nombres naturels t tels qu'à tout nombre premier w corresponde au moins un entier y, non divisible par w, satisfaisant la condition que $t - \psi(y)$ ne soit pas divisible par w. Je distingue trois cas:

I. Supposons que t appartienne à E et que w soit un facteur premier de G. De la définition de l'ensemble E il suit qu'il existe au moins un entier y, non divisible par w, tel que $t - \psi(y)$ ne soit pas divisible par w.

II. Supposons que t appartienne à E et que le nombre premier w ne soit pas un facteur de G. Alors les nombres $\psi(x)$, où x est entier et non divisible par w, n'appartiennent pas à la même classe de restes (mod. w), de sorte qu'on peut trouver au moins un entier y, non divisible par w, tel que $t - \psi(y)$ ne soit pas divisible par w.

III. Supposons que le nombre naturel t n'appartienne pas à E. Dans ce cas au moins un facteur premier w de G possède la propriété que $\psi(x) - t$ soit divisible par w pour tout entier x, non divisible par p. Dans ce cas il est donc impossible de trouver un entier y, non divisible par w, tel que $t - \psi(y)$ ne soit pas divisible par w.

E est donc formé par les nombres naturels t tels qu'à tout nombre premier w corresponde au moins un entier y, non divisible par w, satisfaisant à la condition que $t - \psi(y)$ ne soit pas divisible par w. La proposition 10 nous apprend:
presque tout nombre, appartenant à E, peut être mis sous la forme $p + \psi(p')$, où p et p' sont des nombres premiers.

Retournons à la proposition 10. Pour la démonstration de cette proposition il suffit de traiter le cas particulier où $\psi(x)$ est un polynome à coefficients entiers. Afin de démontrer cette remarque, j'introduis le pro-

duit $G^\star$ des nombres premiers p qui ne sont pas un facteur de U' et pour lesquels la congruence

$$g!\,\psi(x) \equiv g!\,\psi(1) \pmod{p}$$

est vérifiée par chaque entier x qui n'est pas divisible par p. En outre j'introduis l'ensemble $E^\star$ des nombres naturels τ qui satisfont pour tout facteur premier P de $g!KUU'G^\star$ la condition suivante, dans laquelle P^ζ désigne la puissance la plus élevée de P qui est un diviseur de $g!KU$:

1. Si P n'est pas un facteur de U, il existe au moins un entier $X \equiv u' \pmod{U'}$ qui n'est pas divisible par P, tel que $g!\,\psi(X) - \tau$ soit divisible par P^ζ, mais non par $P^{\zeta+1}$.

2. Si P est un facteur de U, il existe au moins un entier $X \equiv u' \pmod{U'}$ qui n'est pas divisible par P, tel que $g!\,\psi(X) + g!Ku - \tau$ soit divisible par P^ζ.

Afin de démontrer que pour tout élément t de E le produit $g!\,t$ appartient à $E^\star$, je distingue trois cas différents:

I. Soit P un facteur premier de $KU'G$, mais pas de U. Au nombre t appartenant à E correspond au moins un entier $X \equiv u' \pmod{U'}$ qui n'est pas divisible par P, tel que $\psi(X) - t$ soit divisible par P^ω, mais non par $P^{\omega+1}$. Alors $g!\,\psi(X) - g!\,t$ est divisible par P^ζ et non par $P^{\zeta+1}$, parce que le facteur P figure dans $g!$ précisément $\zeta - \omega$ fois.

II. Dans le cas, où P est un facteur premier de U, on peut trouver au moins un entier $X \equiv u' \pmod{U'}$ que n'est pas divisible par P, tel que $\psi(X) + Ku - t$ soit divisible par P^ω, de sorte que $g!\,\psi(X) + g!Ku - g!\,t$ est divisible par P^ζ.

III. Soit P un facteur premier de $G^\star$, mais non de $KUU'G$. Alors P^ζ est la puissance la plus élevée de P qui est un diviseur de $g!$ Puisque P n'est un facteur ni de U' ni de G, on peut trouver un entier x non divisible par P tel que $\psi(x) \not\equiv t \pmod{P}$; si l'on choisit $X \equiv u' \pmod{U'}$ et $X \equiv x \pmod{P}$ on obtient donc un entier $X \equiv u' \pmod{U'}$ qui n'est pas divisible par P, tel que $\psi(X) - t$ ne soit pas divisible par P; par conséquent $g!\,\psi(X) - g!\,t$ est divisible par P^ζ et non par $P^{\zeta+1}$.

Si la proposition 10 est déjà démontrée dans le cas particulier où $\psi(x)$ est remplacé par un polynome à coefficients entiers, je puis appliquer cette proposition avec $g!\,\psi(x)$, $g!K$, $G^\star$, $g!N$ et $E^\star$ au lieu de $\psi(x)$, K, G, N et E. Presque tout nombre $g!\,t$ de $E^\star$ peut être mis sous la forme $g!Kp + g!\,\psi(p')$, où p est un nombre premier $\equiv u \pmod{U}$ et p' un nombre premier $\equiv u' \pmod{U'}$, de sorte que presque tout nombre t de E possède la forme (58).

Dans ce qui suit dans cette communication je me bornerai à un polynome $\psi(x)$ dont tous les coefficients sont entiers.

Désignons par $F(t)$ le nombre des manières dont on peut écrire le nombre naturel t sous la forme (58), où $p \equiv u \pmod{U}$ et $p' \equiv u' \pmod{U'}$; posons

$$\Phi(t) = \frac{1}{K\, b^{\frac{1}{g}}\, \varphi(U)\, \varphi(U')} \sum_{h=2b}^{t-2K} \frac{h^{-1+\frac{1}{g}}}{\left(\log \frac{h}{b}\right)\left(\log \frac{t-h}{K}\right)} \quad . \quad . \quad (61)$$

(b est le premier coefficient du polynome $\psi(x) = b x^g + \ldots$; $\Phi(t)$ s'annule pour tout $t < 2K + 2b$) et

$$\Omega_\nu^*(t) = \prod_{p \leqq (\log t)^\nu} W(p, t), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

où $W(p, t)$ est défini pour tout entier t et pour tout nombre premier p de la manière suivante (dans cette définition p^ω est la puissance la plus élevée de p qui est un diviseur de KU, tandis que $\varphi(U)$ désigne le nombre des nombres naturels $\leqq U$ qui sont premiers avec U):

pour un nombre premier p qui n'est pas un facteur de U, le produit

$$\frac{(p-1)\,\varphi(p^{\omega+1} U')}{p^{\omega+1}\,\varphi(U')}\, W(p, t) \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

est le nombre des nombres naturels $h \leqq p^{\omega+1} U'$ qui sont congrus à u' (mod. U') et qui ne sont pas divisibles par p, tels que $\psi(h) - t$ soit divisible par p^ω, mais non par $p^{\omega+1}$;

pour un facteur premier p de U le produit

$$\frac{\varphi(p^\omega U')}{p^\omega\, \varphi(U')}\, W(p, t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

est égal au nombre des nombres naturels $h \leqq p^\omega U'$ qui sont congrus à u' (mod. U') et qui sont premiers avec p^ω, tels que $\psi(h) + Ku - t$ soit divisible par p^ω.

Pour un nombre premier qui n'est un facteur ni de U ni de U', le produit $\frac{(p-1)^2}{p}\, W(p, t)$ est donc égal au nombre des nombres naturels $h \leqq p^{\omega+1}$ qui ne sont pas divisibles par p, tels que $\psi(h) - t$ soit divisible par p^ω, mais non par $p^{\omega+1}$. Pour un nombre premier qui est un facteur de U, mais pas de U', le produit $\frac{p-1}{p}\, W(p, t)$ est égal au nombre des nombres naturels $h \leqq p^\omega$ qui sont premiers avec p^ω, tels que $\psi(h) + Ku - t$ soit divisible par p^ω.

$W(p, t)$ est positif pour tout entier t et pour tout nombre premier p qui n'est pas un facteur de $KUU'G$. En effet, $\frac{(p-1)^2}{p}\, W(p, t)$ est alors le nombre des nombres naturels $h \leqq p-1$ tels que $\psi(h) - t$ ne soit pas divisible par p; la relation $W(p, t) = 0$ impliquerait que p est un facteur de tous les nombres $\psi(x) - t$, où l'entier x n'est pas divisible par p; ce nombre p serait alors un facteur de G.

Pour un facteur premier p de $KUU'G$ il suit de la définition de

l'ensemble E que $W(p,t)$ est positif ou nul, selon que le nombre naturel t appartient à E ou non. Si l'on choisit l'entier $t>1$ et le nombre ν tels que $(\log t)^\nu$ soit supérieur ou égal au plus grand facteur premier de $KUU'G$, le nombre $\Omega_\nu^*(t)$ est donc positif ou nul, selon que t appartient à E ou non.

Pour les entiers $t>1$ qui n'appartienent pas à E, $F(t)$ est, comme nous l'avons observé, borné, et $\varphi(t)\,\Omega_\nu^*(t)$ s'annule, si l'on choisit ν supérieur ou égal au plus grand facteur premier de $KUU'G$.

Proposition 11: *Si m est positif et si l'on a*

$$\nu > 3\,mg + 2m + 2g, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

presque tout nombre t appartenant à E satisfait à la relation

$$F(t) = \left(1 + \frac{\theta}{(\log t)^m}\right) \Phi(t)\,\Omega_\nu^*(t), \text{ où } |\theta| < 1.$$

Le nombre des exceptions $\leqq N$ est pour tout $N \geqq 3$ inférieur à $c_{88} N\,(\log N)^{-m}$, où c_{88} dépend uniquement de m, ν, K, U, U' et du choix du polynome $\psi(x)$.

Il est évident que la proposition 10 est une conséquence immédiate de la proposition 11.

Dans le cas particulier où

$$K = U = U' = 1, \qquad \psi(x) = x^2,$$

G est égal à 6, comme nous l'avons vu, tandis que E est l'ensemble des nombres naturels $t \equiv 0$ ou $\equiv 2 \pmod 6$. Le produit $\frac{(p-1)^2}{p}\,W(p,t)$ est égal au nombre des nombres naturels $h \leqq p-1$ tels que $h^2 - t$ ne soit pas divisible par p, c'est-à-dire $\frac{(p-1)^2}{p}\,W(p,t)$ est pour tout nombre premier $p>2$ égal à $p-3$ ou $p-1$, selon que t est un reste quadratique de p ou non; pour tout nombre pair t on a $\frac{1}{2}W(2,t) = 1$. Ainsi on trouve:

Pour presque tout nombre naturel $t \equiv 0$ ou $\equiv 2$ (mod. 6) le nombre des manières d'écrire t comme un nombre premier augmenté du carré d'un nombre premier, est égal à

$$2\left(1 + \frac{\theta}{(\log t)^m}\right)\left\{\sum_{h=2}^{t-2} \frac{h^{-\frac{1}{2}}}{(\log h)(\log(t-h))}\right\} \Pi_1\left(1 - \frac{p+1}{(p-1)^2}\right) \cdot \Pi_2\left(1 + \frac{1}{p-1}\right)$$

où $|\theta| < 1$; dans cet énoncé je suppose que m soit positif et que ν soit supérieur à $8m+4$; le produit Π_1 est étendu à tous les nombres premiers impairs $p \leqq (\log t)^\nu$, pour lesquels t est un reste quadratique de p, tandis que Π_2 est étendu aux autres nombres premiers impairs $p \leqq (\log t)^\nu$.

Le nombre des exceptions $\leqq N$ est pour tout $N \geqq 3$ inférieur à $c_{89} N\,(\log N)^{-m}$, où c_{89} dépend uniquement de m et de ν.

De la même manière que j'ai déduit la proposition 8 de la proposition 9 (p. 448 et 565), je puis démontrer que la proposition 11 résulte de la proposition suivante:

Proposition 12: *Si M est un nombre positif et si ν est supérieur à gM, on a pour tout $N \geqq 3$*

$$\sum_{t=2}^{[N]} |F(t) - \Phi(t)\,\Omega_\nu^*(t)|^2 < c_{90}\,N^{1+\frac{2}{g}}\,(\log N)^{-M},$$

où c_{90} dépend uniquement de M, ν, K, U, U' et du choix du polynome $\psi(x)$.

La démonstration de cette proposition que je donnerai dans la communication suivante est analogue à celle de la proposition 7 (p. 557—566), mais d'abord je déduirai quelques lemmes. Au lieu du lemme 9 (p. 449) j'utiliserai maintenant le lemme suivant:

Lemme 14: *A tout nombre naturel et à tout polynome*

$$\psi(x) = b\,x^g + \ldots \qquad (b > 0)$$

du degré $g \geqq 1$ à coefficients entiers correspond un nombre positif c_{91} jouissant de la propriété suivante:

Si nous désignons par A' un nombre $\geqq 2b$, par B' un nombre $> A'$, par y un nombre réel, par k un nombre naturel, par h un entier premier avec k et si $\sum_{v' \leqq y}$ est étendu à tous les entiers $v' > A'$ et $\leqq B'$ et $\leqq y$ auxquels correspond au moins un nombre premier $p \equiv h \pmod{k}$ tel que $\psi(p) = v'$, nous avons

$$\left| \sum_{v' \leqq y}^{\star} 1 - \frac{b^{-g}}{\varphi(k)} \sum_{\substack{A' < v' \leqq B' \\ v' \leqq y}} \frac{v'^{-1+\frac{1}{g}}}{\log \frac{v'}{b}} \right| < c_{91}\,B'^{\frac{1}{g}}\,(\log B')^m.$$

Démonstration: Comme nous l'avons vu dans la démonstration du lemme 9 (p. 449) nous pouvons supposer $y \geqq B'$ et $A' \geqq c_{92}$, où c_{92}, dépendant uniquement du choix du polynome $\psi(x)$, est choisi si grand qu'à toute valeur de $v' \geqq c_{92}$ correspond une seule valeur positive de x telle que $\psi(x) = v'$. En choisissant les nombres positifs α et β de telle façon que nous ayons $\psi(\alpha) = A'$ et $\psi(\beta) = B'$, nous obtenons

$$\sum_{v'}^{\star} 1 = \sum_{\substack{\alpha \leqq p \leqq \beta \\ \beta \equiv h \pmod{k},}} 1$$

par suite, en vertu du théorème de SIEGEL–WALFISZ (lemme 7; p. 448)

$$\left| \sum_{v'}^{\star} 1 - \frac{1}{\varphi(k)} \int_\alpha^\beta \frac{du}{\log u} \right| < c_{93}\,\beta\,(\log \beta)^{-m} < c_{94}\,B'^{\frac{1}{g}}\,(\log B')^{-m};$$

c_{93} dépend uniquement de m, tandis que c_{94} dépend seulement de m et du choix du polynome $\psi(x)$.

En posant $w = \psi(u)$, on obtient

$$\int_{\alpha}^{\beta} \frac{du}{\log u} = \int_{A'}^{B'} \frac{dw}{\psi'(v) \log u},$$

où

$$b u^g = w \left(1 + O\left(w^{-\frac{1}{g}}\right)\right),$$

$$u = \left(\frac{w}{b}\right)^{\frac{1}{g}} \left(1 + O\left(w^{-\frac{1}{g}}\right)\right),$$

$$\psi'(u) = g b u^{g-1} (1 + O(u^{-1}))$$

$$= g b^{\frac{1}{g}} w^{1-\frac{1}{g}} \left(1 + O\left(w^{-\frac{1}{g}}\right)\right),$$

$$\log u = \frac{1}{g} \log \frac{w}{b} + O\left(w^{-\frac{1}{g}}\right),$$

par conséquent

$$\int_{\alpha}^{\beta} \frac{du}{\log u} = b^{-\frac{1}{g}} \int_{A'}^{B'} \frac{w^{-1+\frac{1}{g}}\, dw}{\log \frac{w}{b}} + O \int_{A'}^{B'} \frac{w^{-1}\, dw}{\log \frac{w}{b}}$$

$$= b^{-\frac{1}{g}} \sum_{A' < v' \leqq B'} \frac{v'^{-1+\frac{1}{g}}}{\log \frac{v'}{b}} + O(\log \log (3 + B')).$$

Ainsi le lemme 14 est démontré.

En outre j'ai besoin du théorème suivant, dû à M. I. M. VINOGRADOW.

Lemme 15: *A chaque couple de nombres naturels g et m correspondent un nombre naturel η et un nombre positif c_{95} ayant la propriété suivante:*

Si Z est supérieur à 3 et si $\chi(x) = \alpha x^g + \ldots$ désigne un polynome réel du degré g, tandis qu'à α correspond une fraction irréductible $\frac{a}{q}$ telle que

$$\left| \alpha - \frac{a}{q} \right| \leqq q^{-1} Z^{-g} (\log Z)^{\eta} \quad \text{et} \quad (\log Z)^{\eta} \leqq q \leqq Z^g (\log Z)^{-\eta},$$

on a

$$\left| \sum_{p \leqq Z} e^{2\pi i \chi(p)} \right| < c_{95} Z (\log Z)^{-m} \ldots \ldots \ldots \quad (66)$$

Lemme 16: *Les nombres η et c_{95}, définis dans le lemme précédent, possèdent la propriété suivante: pour toute paire de nombres réels $Z > 3$ et α, telle que l'intervalle fermé $(\alpha \mp Z^{-g}(\log z)^{\eta})$ ne contienne aucune fraction à dénominateur positif $< (\log Z)^{\eta}$, chaque polynome réel $\chi(x) = \alpha x^g + \ldots$ du degré g satisfait l'inégalité (66).*

Dans ces communications $(\alpha \mp \zeta)$ désigne l'intervalle $(\alpha - \zeta,\ \alpha + \zeta)$.

Démonstration: Posons $\tau = Z^g (\log Z)^{-\eta}$.

Puisque l'intervalle fermé $\left(\alpha \mp \frac{1}{\tau}\right)$ ne contient aucun nombre entier, on a $\tau > 1$. D'après un théorème connu de DIRICHLET, il existe au moins une fraction irréductible $\frac{a}{q}$ telle qu'on ait

$$0 < q \leqq \tau \quad \text{et} \quad \left| \alpha - \frac{a}{q} \right| < \frac{1}{q\tau}.$$

Cette fraction $\frac{a}{q}$ étant située dans l'intervalle $\left(\alpha \mp \frac{1}{\tau}\right)$ possède un dénominateur $\geqq (\log Z)^{\eta}$, de sorte que le lemme 16 découle du lemme précédent.

Lemme 17: *A chaque système de nombres naturels g, m et U' correspondent un nombre naturel ζ et un nombre positif c_{96} ayant la propriété suivante: pour toute paire de nombres réels $Z > 3$ et α, telle que l'intervalle fermé $(\alpha \mp Z^{-g}(\log z)^{\zeta})$ ne contienne aucune fraction à dénominateur positif $< (\log Z)^{\zeta}$, chaque polynome réel $\chi(x) = \alpha x^g + \ldots$ du degré g satisfait pour tout entier u', qui est premier avec U', l'inégalité*

$$\left| \sum_{\substack{p \leqq Z \\ p \equiv u' \,(\mathrm{mod.}\ U')}} e^{2\pi i \chi(p)} \right| < c_{96} Z (\log Z)^{-m}.$$

Démonstration: 1. Soit $g = 1$. Le nombre $\zeta = \eta + 1$, où η désigne le nombre mentionné dans le lemme 15, possède la propriété désirée. En effet, sans nuire à la généralité nous pouvons supposer $\log Z > U'$. Soit h un entier quelconque. Puisque $(\alpha \mp Z^{-g}(\log z)^{\eta+1})$ ne contient aucune fraction à dénominateur positif $< (\log Z)^{\eta+1}$, l'intervalle fermé $\left(\alpha + \frac{h}{U'} \mp Z^{-g}(\log Z)^{\eta}\right)$ ne renferme aucune fraction à dénominateur positif $< \frac{(\log Z)^{\eta+1}}{U'}$, par suite à plus forte raison aucune fraction à dénominateur positive $< (\log Z)^{\eta}$. Le lemme précédent nous apprend donc

$$\left| \sum_{p \leqq Z} e^{2\pi i \left(\alpha + \frac{h}{u'}\right) p} \right| < c_{95} Z (\log Z)^{-m},$$

d'où il suite que

$$\sum_{\substack{p \leqq Z \\ p \equiv u' \,(\mathrm{mod.}\ U')}} e^{2\pi i \alpha p} = \frac{1}{U'} \sum_{h=1}^{U'} e^{\frac{-2\pi i h u'}{U'}} \sum_{p \leqq Z} e^{2\pi i \left(\alpha + \frac{h}{U'}\right) p}$$

est également en valeur absolue inférieure à $c_{95} Z (\log Z)^{-m}$.

2. Soit $g \geqq 2$. Le nombre $\zeta = \eta$ défini dans le lemme 15 possède la propriété désirée. Car d'après le lemme précédent, appliqué avec $\chi(x) + \frac{hx}{U'}$ au lieu de $\chi(x)$, on a

$$\left| \sum_{p \leqq Z} e^{2\pi i \left(\chi(p) + \frac{hp}{U'}\right)} \right| < c_{95} Z (\log Z)^{-m},$$

d'où il suit que

$$\sum_{\substack{p \leqq Z \\ p \equiv u' \pmod{U}}} e^{2\pi i \chi(p)} = \frac{1}{U'} \sum_{h=1}^{U'} e^{-\frac{2\pi i h u'}{U'}} \sum_{p \leqq Z} e^{2\pi i \left(\chi(p) + \frac{hp}{U'}\right)}$$

est également en valeur absolue inférieure à $c_{95} Z (\log Z)^{-m}$.

Lemme 18: *A chaque système de nombres naturels g, m, U' et b correspondent un nombre naturel η et un nombre positif c_{97} ayant la propriété suivante: pour toute paire de nombres réels $Z > 3$ et α, telle que l'intervalle fermé $(\alpha \mp Z^{-g} (\log Z)^{\eta})$ ne contienne aucune fraction à dénominateur positif $< (\log Z)^{\eta}$, chaque polynome réel $\chi(x) = \alpha b x^g + \ldots$ du degré g satisfait pour tout entier u', à l'inégalité*

$$\left| \sum_{\substack{p \leqq Z \\ p \equiv u' \pmod{U'}}} e^{2\pi i \chi(p)} \right| < c_{97} Z (\log Z)^{-m}. \quad . \quad . \quad . \quad . \quad . \quad (67)$$

Démonstration: Le nombre $\eta = \zeta + 1$, où ζ désigne le nombre mentionné dans le lemme précédent, possède la propriété désirée. En effet, nous pouvons supposer $\log Z > b$; puisque $(\alpha \mp Z^{-g} (\log Z)^{\zeta+1})$ ne contient aucune fraction à dénominateur positif $< (\log Z)^{\zeta+1}$, l'intervalle fermé $(\alpha b \mp Z^{-g} (\log Z)^{\zeta})$ ne renferme aucune fraction à dénominateur positif $< \frac{(\log Z)^{\zeta+1}}{b}$, par suite à plus forte raison aucune fraction à dénominateur positif $< (\log Z)^{\zeta}$, de sorte que le lemme 18 est une conséquence immédiate du lemme précédent.

Lemme 19: *A chaque système de nombres naturels g, m, U' et b correspondent un nombre naturel ζ et un nombre positif c_{98} ayant la propriété suivante: pour tout système de nombres réels $Z > 3$, Y et α, tel que l'intervalle fermé $(\alpha \mp Z^{-g} (\log Z)^{\zeta})$ ne contienne aucune fraction à dénominateur positif $< (\log Z)^{\zeta}$, chaque polynome réel $\chi(x) = \alpha b x^g + \ldots$ du degré g satisfait, pour tout entier u' qui est premier avec U', à l'inégalité*

$$\left| \sum_{\substack{Y \leqq p \leqq Z \\ p \equiv u' \pmod{U'}}} e^{2\pi i \chi(p)} \right| < c_{98} Z (\log Z)^{-m}.$$

Démonstration: Le nombre $\zeta = 1 + \eta + g m$, où η est le nombre figurant dans le lemme précédent, possède la propriété désirée. En effet, l'inégalité

(67) est alors valable, de sorte que le lemme à démontrer est évident, si $Y \leqq Z\,(\log Z)^{-m}$. Supposons

$$Y > Z\,(\log Z)^{-m}, \text{ par suite } Y^{-g}\,(\log Y)^{\eta} \leqq Z^{-g}\,(\log Z)^{\xi}$$

si Z est suffisamment grand (sinon le lemme est évident). L'intervalle fermé $(\alpha \mp Y^{-g}(\log Y)^{\eta})$ ne contient donc aucune fraction à dénominateur $< (\log Y)^{\eta}$, et le lemme précédent (appliqué avec Y au lieu de Z) nous apprend

$$\Big|\sum_{\substack{p<Y \\ p \equiv u' \,(\mathrm{mod.}\, U')}} e^{2\pi i \alpha \chi(p)}\Big| < c_{97}\, Y\,(\log Y)^{-m} < c_{99}\, Z\,(\log Z)^{-m}.$$

Ainsi le lemme 19 est démontré.

Je donnerai la démonstration de la proposition 12 dans la communication suivante.

Plantkunde. — *Snelle bloei van Hollandsche Irissen.* II. (Avec résumé.) Door A. H. Blaauw, Ida Luyten en Annie M. Hartsema. (Mededeeling No. 57 van het Laboratorium voor Plantenphysiologisch Onderzoek te Wageningen.)

(Communicated at the meeting of December 17, 1938.)

Voortgezet werden de onderzoekingen omtrent het snel in bloei brengen van de Hollandsche Bol-iris „Imperator". Wij hebben daarbij een hooge niet langdurende *vóór-temperatuur* (boven 20°) toegepast, gevolgd door een langdurende lage *prepareer-temperatuur* (meestal 9°, soms 7°). *Tijdens* deze hooge vóór-temperatuur (bijv. 23° of 28° of 31°) staat de groei zoowel als de vorming van organen vrijwel stil. Maar deze vóór-temperatuur heeft, ook al duurt ze kort, een begunstigenden invloed op de *dispositie* voor den bloemaanleg, die pas veel later in de kou plaats vindt. Te lang moet die hooge temperatuur weer niet duren, daar anders de latere bloemaanleg en daarmee ook de bloei veel te veel verschoven wordt. Direct na het rooien de prepareerende koele temperatuur geven, zou het snelste werken, maar dit bleek het bloeivermogen te zeer te reduceeren.

De vroeger beschreven proeven van 1934—'35 en 1935—'36 hadden als voorloopig resultaat het volgende opgeleverd.

Een vóór-temperatuur van 1 week 31° (of 28°) werd gevolgd door een prepareer-temperatuur van 9°, waarbij direct in kistjes werd geplant; deze prepareering werd voortgezet tot het loof minstens 6 cm buiten de bollen was. Deze behandeling duurt tot dat tijdstip ± 70 dagen. Daarna werden de bollen in 15° in bloei gebracht, wat nog ruim 80 dagen duurt na 6 cm-looflengte. Deze methode gaf het vlugste resultaat, maar is iets meer geforceerd, daar slechts 1 week hooge vóór-temperatuur werd gegeven. Hiervoor moeten bepaald zeer zware goed gevoede bollen gebruikt worden.

Veiliger maar wat langduriger bleek het te zijn 5 weken 23° te geven en dan bij een prepareer-temperatuur van 7° te planten. Wij kregen op deze wijze in 1936 de 20 bollen van deze proef alle in bloei op 16 tot 21 Februari.

Intusschen waren verschillende punten in deze proeven nog onbevredigend, want voor bloei in Januari bleek de kans op mislukken te groot. Maar vooral niet bevredigend, omdat de maat (6 cm lengte van de langste der eerste 3 à 4 bladen buiten den bol), waarbij de planten uit de prepareer-temperatuur naar de trek-temperatuur van 15° C. overgingen, een te willekeurig probeeren is.

Is de bloem dan reeds in 9° (of 7°) aangelegd? Is het noodzakelijk dat de bloem in die lagere temperaturen gevormd is, vóór de bollen naar de hoogere trek-temperatuur gaan? Ligt hierin soms de oorzaak, dat de trek

van Irissen bij een zoo lage temperatuur (niet boven 15° C.) moet plaats hebben, zooals de kweeker uit ervaring weet? Of kan de bloem ook in 15° C. worden afgemaakt, mits de aanleg tot een zeker stadium is ingeleid? In Mededeel. No. 48, bl. 6 (Proceedings p. 609) hebben we er reeds op gewezen, dat mislukking optreedt, als in de koele temperatuur de bloemaanleg niet reeds ten deele begonnen is. Verder hebben we vroeger reeds gevonden, dat in bollen, liggende in temperaturen van 13° C. en hooger, de bloemvorming steeds moeilijker plaats heeft. Bij 20° C. is bloemvorming onmogelijk, in 17° C. reeds zeer twijfelachtig.

Wij herhalen hier enkele van deze zoo merkwaardige feiten, omdat juist hierdoor de cultuur, en daarbij ook het vroeg-bloeien, bijzondere moeilijkheden oplevert. Tot dusver is ons nog geen tweede plantengroep bekend, die zoo lage temperaturen voor de bloemvorming vereischt. En bij den trek bestaat dus de moeilijkheid daarin, dat de bloemaanleg nog moet plaats vinden als de prepareerende behandeling voor den vroegen bloei (d.w.z. voor een vlugge strekking) al in vollen gang is.

In '36—'37 werd nu bij 6 cm looflengte overgebracht, maar ook na nog 2 weken en 4 weken de lage temperatuur te hebben voortgezet; in '37—'38 werd dit alleen gedurende 3 weken gedaan. Bij het overbrengen naar 15° C. werd de toestand gefixeerd en later onderzocht. Beide jaren kunnen wij in dat opzicht samenvoegen om een beeld te krijgen van den toestand bij den overgang van 9° (event. 7°) naar 15° C. Zie tabel 1 en 2.

Blijven wij eerst bij deze 2 tabellen, dan is het bij deze voorstellingswijze zonder meer duidelijk, dat bij den overgang uit 9° naar de trek-tempe-

TABEL 1. Vóór-temperatuur 31° C. Prepareer-temperatuur 9° (direct geplant). Stadium bij overgang van 9° in de trek-temperatuur 15° C.

	Naar kas 15° na looflengte:	I	I—II	II–	II	II+	II—III	III–	III	III+	III—IV	IV–	IV	IV+	IV—V	V–	V	V+
	6 cm (= 29 Oct. '36 en 5 Nov. '37)		2	2	1	2	0	2										
	6 cm + 2 w. 9° licht				1	0	0	1	0	1	5	2						
Bollen van *kleigrond*	6 cm + 3 w. 9° licht 1937						1	2	1	5	0	1						
Bollen van zandgrond	6 cm + 3 w. 9° licht 1937									3	2	2	1	1	1			
evenals alle andere	6 cm + 4 w. 9° licht												1	1	0	4	3	

TABEL 2. Vóór-temperatuur 31° C. Prepareer-temperatuur 9°, waarvan de eerste 6 weken droog (ongeplant).

Naar kas 15° na looflengte:		I	I—II	II	II	II+	II—III	III−	III	III+	III—IV	IV−	IV	IV+	IV—V	V−	V	V+
6 cm (= 4 Nov. '36 en 10 Nov. '37)			3	3	0	2	1	1										
6 cm + 2 w. 9° licht								1	0	1	5	2						
6 cm + 3 w. 9° licht	1937									1	0	2	4	2	1			
6 cm + 4 w. 9° licht															1	2	6	1

ratuur (15°), de bloemvorming in vollen gang is. Verder, dat hoe langer men na 6 cm in 9° laat, deze des te meer is voortgeschreden.

Daar 15° geen gunstige temperatuur meer is, neemt de risico af, wanneer men later naar 15° overbrengt, daar de bloem dan veel verder gevorderd is, maar men verliest natuurlijk eenige dagen of meer in snelheid van bloeien.

Wij willen er hier reeds op wijzen, dat de bollen van kleigrond wel 7 dagen later (pas 25 Aug. '37) ontvangen werden dan de overige zandbollen, maar dat ze op denzelfden tijd 6 cm bereikten en op een gelijk moment, d.w.z. 3 weken later (26 Nov.), in bloemvorming *minder ver gevorderd* zijn dan de gelijk behandelde zandbollen. Straks komen wij hierop nog verder terug.

In 1936—'37 waren de proeven geheel gebaseerd op de voorbehandeling 1 week 31° enz., die in '35—'36 zoo gunstige resultaten hadden gegeven (80 % bloei in Januari). Maar thans waren die uitkomsten zoo slecht, dat in '37—'38 naast deze behandeling, die toch als riskant beschouwd moet worden, *tevens* de kalmere en gunstige behandeling werd toegepast met een vóór-temperatuur van 5 weken 23°—25½°—28° of 31°. De einduitkomst wordt nader besproken. Hier volgt eerst tabel 3 om te laten zien, hoe het staat met den bloemaanleg, wanneer deze andere behandeling gegeven wordt. Nu werd in dit laatste jaar (1937—'38), nadat 6 cm looflengte bereikt was, de koude (9° en 7°) *steeds* nog 3 weken in licht met 9° *voortgezet* en wel veiligheidshalve.

Uit deze tabel blijkt:

1°. De lengte van 6 cm loof wordt na verschillende vóór-temperaturen (23° tot 31°) op denzelfden tijd bereikt (zie 3e kolom).

2°. De prepareer-temperatuur 7° of 9° heeft echter grooten invloed op

TABEL 3. Vóór-temperatuur 5 weken 23° tot 31° C. Prepareer-temperatuur 9° of 7°. Stadium bloemaanleg 3 weken na 6 cm looflengte.

Vóór-temperatuur	Prepareer-temperatuur	Datum 6 cm loof	Stadium bloemaanleg 3 weken later																			
			I	I—II	II–	II	II+	II—III	III–	III	III+	III—IV	IV–	IV	IV+	IV—V	V–	V	V+	V—VI	VI–
5 w. 23°	9°	25 Nov.							10													
5 w. 23° (klei)	9°	27 Nov.				1	0	1	3	3												
5 w. 23°	7°	13 Dec.																8	0	1		
5 w. 25½°	9°	25 Nov.						5	5													
5 w. 25½°	7°	13 Dec.															1	9				
5 w. 28°	9°	25 Nov.								2	7											
5 w. 28° (klei)	9°	27 Nov.					2	3	2	1												
5 w. 28°	7°	13 Dec.														1	2	6	0	1		
5 w. 31°	9°	25 Nov.					1	0	0	7	2											
5 w. 31°	7°	13 Dec.													1	1	1	7				

het bereiken van deze looflengte, daar deze in 7° pas *16 dagen later* bereikt wordt dan in 9° C.

3°. Ook na een warme vóórbehandeling van 5 weken is de bloemvorming 3 weken na 6 cm, dus bij 't overbrengen van 9° naar 15° in vollen gang, maar over 't algemeen een weinig minder ver (gemidd. III– tot III) dan na 1 week 31° (gemidd. III à IV tot IV), zie tab. 1 en 2 bij 3 w. na 6 cm.

4°. Na 1 week vóórwarmte (tab. 1 en 2) in plaats van 5 weken (tab. 3) wordt 6 cm ongeveer 3 weken vroeger bereikt (resp. 5 en 25 Nov.).

5°. Terwijl onder 2°. vermeld wordt, dat bij een prepareer-temperatuur van 7° de *strekking* tot 6 cm-lengte pas 16 dagen later bereikt wordt dan in 9°, toont tabel 3 verder, dat de *bloemaanleg* bij een overeenkomstige lengte — maar dan ook ± 16 dagen later — in 7° aanzienlijk *meer gevorderd* is dan in 9° (Stad. V tegenover ± III). Dit punt is van veel belang voor deze forceerproeven, want wij hebben reeds meer aanwijzingen, dat 7° onder bepaalde omstandigheden voor den bloemaanleg van beteekenis is (zie ook reeds in Mededeel. No. 48). De strekking der bladen gaat dus na een warme vóórbehandeling in 7° duidelijk langzamer dan in 9°, de bloemaanleg gaat waarschijnlijk bijna even snel.

6°. Bollen van kleigrond zijn in bloemaanleg na 5 w. 23° niet-, na 5 w. 28° een weinig achter bij de bollen van zandgrond, die 2 dagen vroeger konden worden overgebracht en gefixeerd.

7°. Bovendien werden proeven genomen met bollen, die in de koude

niet direct werden geplant, maar eerst 6 of 9 weken droog bleven liggen (slechts een deel van deze proeven is in tabel 2 vermeld).

Wanneer na 1 w. 31° C. *direct* in kou geplant werd, of 6 weken of 9 weken eerst droog bewaard in 9°, dan werd in 1936—'37 de 6 cm looflengte resp. op 29 Oct., 4 Nov. en 16 Nov. bereikt. Het overbrengen uit 9° in 15° 't zij bij 6 cm, òf 2 of 4 weken later, werd dus door het droog koelen merkbaar vertraagd, vooral boven 6 weken. Het stadium van bloemaanleg was na 6 weken droog liggen bij den overgang naar 15° C. heel weinig verder, doordat dit 6 dagen later geschiedde (verg. bijv. tabel 1 en 2); na 9 weken droog liggen — wat in de looflengte 18 dagen vertraging gaf — was het stadium (18 dagen later) iets verder dan na direct planten. Wij hebben van eerst droog koelen gedurende 6 weken ook wat den bloei betreft geen voordeelen gezien; het gaf ongeveer een week verlating in den bloei, maar verder geen nadeel. Is het in de praktijk met het oog op verzending van te trekken bollen gewenscht, *dan kan men gerust 6 weken droog koelen, maar wij raden af dit langer dan 6 weken te doen,* daar de verlating van den bloei dan aanzienlijk gaat toenemen.

Alles samen genomen weten wij dus dat na verschillende warme vóórbehandelingen, gevolgd door korter of langer prepareer-koude van 9° of 7°, de bollen bij den overgang van de koude naar 15° C. midden in den bloemaanleg zijn. Dat het stadium natuurlijk verder gevorderd is, als men langer in de koude laat (2 of 3 of 4 weken na 't bereiken van 6 cm loof). En ten slotte dat na behandeling met koude van 7° het stadium verder gevorderd is bij gelijke looflengte dan na 9°, doordat diezelfde lengte pas 16 dagen later werd bereikt.

Uit een en ander zal het duidelijk zijn, waardoor de vroege bloei van Bol-irissen zooveel moeilijkheden oplevert.

Hieronder volgen nu de uitkomsten.

In het jaar 1936—'37 waren de proeven geheel gebaseerd op 1 week 31°, gevolgd door 9° en dan òf bij ruim 6 cm looflengte naar de trekkas van 15°, of 2 of 4 weken later, waarbij de kistjes gedurende die 2 of 4 weken in 9° in een licht kasje op het noorden stonden. Alle hoop was toen gevestigd op het verder gereed zijn van de bloem, alvorens de hoogere temperatuur werd toegepast. Vermoed werd, dat een te vroeg overbrengen de meeste mislukking veroorzaakte, doordat de bloem niet ver genoeg gereed was om 15° C. te kunnen verdragen. Ofschoon dit in 't algemeen ook juist is, zoo gaf dit jaar een volkomen mislukking. Slechts in enkele proeven kwamen een paar bloemen tot haar recht. Zoodoende viel er niets af te leiden uit de proeven van dit jaar; zoo lag er ook geen bewijs in, dat een verder aanleggen van de bloem vóór het overbrengen naar 15° C. gunstiger werkte. Toch moest hieraan worden vastgehouden. Eveneens was het moeilijk te gissen naar de oorzaak van dit algemeen mislukken. Van belang is het vooral er op te wijzen, dat bij deze behandelingen *overal bloem-aanleg plaats vond,* zoodat daarin de fout niet kon liggen.

Wellicht kan deze daaraan gelegen hebben, dat de bollen bij ontvangst gemiddeld iets lichter waren (12 tot 18 g) dan het vorige jaar (14 tot 21 g). Vooral ook, omdat uitsluitend met 1 week 31° werd vóórbehandeld, wat den snellen bloei kan bevorderen, en niet met 5 weken 23° à 28°, wat sterk verlaat, maar het bloeivermogen doet stijgen. Daarom werden in 1937—'38 weer beide methoden toegepast en de uitkomst daarvan gaf wel het bewijs, dat deze overweging omtrent het mislukken waarschijnlijk de juiste is. De opzet en de uitslag van deze nieuwe proeven volgen hieronder. Het gewicht der bollen bedroeg van 14 tot 20 gram (9—10 cm omtrek).

De eene groep proeven had als basis weer 1 week 31° vóór-warmte. Daarbij werd beproefd of betere voeding een beter resultaat kon bewerken. Zoo werden bollen van kleigrond vergeleken met bollen van zandgrond (beiden 14—20 g wegende). De eerste 3 tot 4 bladen verschijnen reeds vroeg, lang voor men aanwijzing heeft (bij het verschijnen van 't 5e blad) dat er bloem kan komen. Er was veel kans, dat in die weken de assimilatie van de eerste bladen en in 't algemeen ook de voeding uit den bodem nog van beteekenis kon zijn voor het vormen en gelukken der bloemen. Daarom werd een proef met Neon-licht toegevoegd (2 klein model buizen, type 4312; 't midden 60 cm links en rechts boven het midden der kistjes; gedurende 8 uur na 't daglicht; beginnende 27 Nov. tot 21 Jan., als de knoppen omhoog komen). In de meeste proeven werd de grond (duinzand) met fijnkorrelige gravel luchtiger gemaakt; in een paar proeven

TABEL 4.

Vóór-temperatuur	Verdere omstandigheden	Geplant in 9°	Met 6 cm in licht in 9° gedur. 3 w.	Na 3 w. in 15°	1e bloem open	Laatste bloem	Aantal in bloei van 30 stuks
1 w. 31°	klei-bollen in zand met gravel	2 Sept.	5 Nov.	26 Nov.	10 Febr.	17 Febr.	23
1 w. 31°	zand-bollen in zand met gravel	25 Aug.	5 Nov.	26 Nov.	10 Febr.		3
1 w. 31°	in zand + voedingsopl. en gravel	25 Aug.	5 Nov.	26 Nov.	8 Febr.	14 Febr.	11
1 w. 31°	in zand zonder gravel	25 Aug.	5 Nov.	26 Nov.	8 Febr.		2
1 w. 31°	in zand zonder gravel, met voedingsopl.	25 Aug.	5 Nov.	26 Nov.	6 Febr.	11 Febr.	6
1 w. 31°	in zand + Neonlicht	25 Aug.	5 Nov.	26 Nov.	4 Febr.	12 Febr.	24
1 w. 31°	eerst 6 w. 9° droog	6 Oct.	10 Nov.	1 Dec.	13 Febr.		1

werd de grond in de kistjes voedzamer gemaakt door toevoeging van v. d. Crone's oplossing (50 cc per week per kistje). Zie verder tabel 4.

In alle proeven is dit jaar voor veiligheid na 6 cm looflengte de lage prepareer-temperatuur nog 3 weken voortgezet, opdat de bloemvorming verder gevorderd zou zijn bij het overbrengen in 15° C. (zie Tab. 1 en 2).

Het is nu wel zeer opvallend, dat hier betere voeding, 't zij tevoren of gedurende de proeven, het gelukken bepaalt.

Bollen van kleigrond bloeiden voor ruim 75 %; bollen waarvan 't loof met Neon-licht werd bestraald voor 80 %; bollen in duinzand met voedingsoplossing en gravel voor ruim 35 %; bollen in duinzand met voedingsoplossing, zonder gravel nog voor 20 %, terwijl zonder gunstige voedingsomstandigheden hoogstens een paar bloemen van de 30 gelukken.

Wil men dus trachten de Irissen zeer vroeg in bloei te hebben door te beginnen met slechts 1 week 31° C., dan gebruike men *zoo zwaar mogelijke bollen die op kleigrond hebben gestaan* en verder *goeden voedzamen zandgrond in de kistjes*. Wie er de gelegenheid toe heeft, zal ook door *Neon-bestraling* het resultaat kunnen verbeteren. Beginnen met enkele weken droog te koelen zal bij deze geforceerde behandeling vermeden moeten worden.

In de tweede plaats volgen die proeven, waarbij de trek veiliger, maar ook langzamer werd gemaakt door langer een warme vóórbehandeling te geven. Daarbij werden als overgang en vergelijking behalve 1 w. 31° ook 2 en 3 en 5 w. 31° C. onderzocht. In tabel 5 wordt ook 1 w. 31° uit tab. 4 daartoe herhaald. Deze proeven werden alle met bollen van zandgrond uitgevoerd, behalve een paar proeven ter vergelijking die van kleigrond kwamen; alles werd in duinzand met gravel geplant zonder extra voedingsmiddelen.

Uit tabel 5 volgen deze conclusies:

1°. Reeds de serie 1—2—3 of 5 w. 31° doet zien, dat een langere warme vóórbehandeling het aantal slagende bloemen doet toenemen; maar tevens dat 31° voor die lange vóórwarmte te hoog is (tot 63 % bloeiers).

2°. De tien vermelde vóórbehandelingen met 5 w. 23° of 25½° of 28° *geven ruim 80 % tot 100 % bloeiers.*

3°. Na het toepassen van 5 weken vóór-warmte gedragen de bollen *van zandgrond zich even gunstig als van kleigrond*; terwijl toch bij snel forceeren met slechts 1 week vóórwarmte de bollen van kleigrond zeer goede, van zandgrond zeer slechte resultaten gaven.

4°. Bij deze behandeling kan men wel 6 weken eerst droog koelen in 9°, het geeft alleen enkele dagen verlating.

5°. *Zeer gunstig is na 5 w. 25½° of 23° een prepareerkoude van 7° C.*, waarbij resp. 95 % en 100 % bloeit. Deze uitkomst is des te meer betrouwbaar, daar ook in 1935—'36 na 5 w. 23° gevolgd door 7° C. 100 % bloeiers werden verkregen. Destijds werden de bollen op 7 Aug. reeds ontvangen; dit laatste jaar pas 18 Aug. Bovendien werd dit jaar 3 weken langer in

TABEL 5.

Vóór-temperatuur	Verdere omstandigheden	Geplant in	Met 6 cm in licht in 9° gedur. 3 w.	Na 3 w. 15° C.	1e bloem open	Laatste open	Aantal in bloei van 30 stuks
1 w. 31°		9° op 25 Aug.	5 Nov.	26 Nov.	10 Febr.		3
2 w. 31°		9° op 1 Sept.	8 Nov.	29 Nov.	14 Febr.		3
3 w. 31°		9° op 8 Sept.	12 Nov.	3 Dec.	19 Febr.	25 Febr.	8
5 w. 31°		9° op 22 Sept.	25 Nov.	16 Dec.	4 Mrt.	7 Mrt.	19
5 w. 28°		9° op 22 Sept.	25 Nov.	16 Dec.	3 Mrt.	8 Mrt.	25
5 w. 28°	van kleigrond	9° op 30 Sept.	27 Nov.	18 Dec.	8 Mrt.	14 Mrt.	29
5 w. 28°	eerst 6 w. 9° droog	9° op 3 Nov.	3 Dec.	24 Dec.	9 Mrt.	15 Mrt.	28
5 w. 28°		7° op 22 Sept.	13 Dec.	3 Jan.	9 Mrt.	17 Mrt.	26
5 w. 25½°		9° op 22 Sept.	25 Nov.	16 Dec.	3 Mrt.	7 Mrt.	27
5 w. 25½°		7° op 22 Sept.	13 Dec.	3 Jan.	6 Mrt.	15 Mrt.	29
5 w. 23°		9° op 22 Sept.	25 Nov.	16 Dec.	27 Febr.	6 Mrt.	28
5 w. 23°	van kleigrond	9° op 30 Sept.	27 Nov.	18 Dec	3 Mrt.	10 Mrt.	25
5 w. 23°	eerst 6 w. 9° droog	9° op 3 Nov.	6 Dec.	3 Jan.*)	11 Mrt.	18 Mrt.	29
5 w. 23°		7° op 22 Sept.	13 Dec.	3 Jan.	7 Mrt.	11 Mrt.	30

*) Door vergissing 4 i. pl. v. 3 w. nog in 9° gelaten.

9° C. gelaten na 6 cm looflengte. Daardoor viel dit jaar de bloei 20 dagen later (7 Mrt.) dan in 1935—'36 (16 Febr.), maar overigens met hetzelfde resultaat.

Voor een veilig trekken kunnen wij dus blijven aanbevelen *5 weken 23° gevolgd door 7°*, liefst direct geplant. In een volgend jaar zal nader worden nagegaan of het noodzakelijk is na 6 cm looflengte langer 7° te blijven geven met het oog op den bloemaanleg. Volgens '35—'36 en '37—'38 maakte dit geen verschil uit, en gaf alleen tijdverlies. Dat komt doordat

immers 7° langzamer strekt dan 9°, maar dan ook bij 6 cm looflengte met de bloemvorming in Stad. V is tegenover Stad. III bij 6 cm looflengte na 9°. Aldus werd dan ook in '35—'36 na 5 w. 23° gevolgd door 9° slechts ± 65 % bloei bereikt, maar gevolgd door 7° de volle 100 %.

Uit een en ander volgt dus in het algemeen:

1°. Dat 5 weken vóór-warmte het bloeivermogen sterk bevordert.

2°. Dat het gewenscht is, dat de bollen niet te vroeg uit de prepareerkoude in 15° C. komen; dat hierbij niet de looflengte het tijdstip bepaalt, maar het stadium, waarin de bloemvorming verkeert (gunstig is bijv. ± Stadium V).

3°. Dat bij sterk forceeren, na 1 week 31°, de voeding van de bollen, te voren of (resp. èn) tijdens de proef een groote rol speelt.

Wageningen, November 1938.

RÉSUMÉ.

Floraison précoce des Iris hollandais. II.

L'étude de l'obtention d'une floraison précoce de l'Iris hollandais „Imperator" a été poussée davantage. Une *température préalable* élevée (par ex. 23°, 28° ou 31°) et de courte durée favorise les dispositions de la préformation d'une fleur qui ne se produira que beaucoup plus tard dans le froid. Puis vient une *température de préparation,* basse et de longue durée, le plus souvent 9°, parfois 7°, qui permet que plus tard, après bien des semaines, on procède à un lent forçage des plantes (à une température voisine de 15°).

Nombreuses sont les autres plantes bulbeuses qui ont déjà formé leur fleur tout à fait ou en grande partie au moment où on les plante dans les boîtes. La difficulté du forçage des Iris bulbeux réside justement dans ce que la formation de leur fleur ne se produit que très tard, de sorte qu'elle doit encore avoir lieu dans le froid alors qu'on procède déjà à la préparation des bulbes pour la floraison précoce. La formation de la fleur se produit très bien à froid à 9° et à 7°, mais risque d'échouer dès 15° et au-dessus. Ainsi convient-il que, pendant la seconde moitié du traitement par le froid, la préformation de la fleur soit déjà achevée dans ses grandes lignes avant le transfert à 15°. Nous avons maintenant examiné de plus près l'évolution de cette formation florale au cours du traitement pour assurer un plus grand succès de la floraison précoce.

Autrefois nous donnions 15° de température dès que la longueur des feuilles atteignait 6 cm; cette fois-ci (1936 et 1937), nous ne l'avons fait que 2, 3 et 4 semaines après avoir atteint cette longueur. Le tabl. 1 montre le progrès de la formation florale à 6 cm de longueur de feuille, puis respectivement avec 2—3—4 semaines (1e, 2e, 3e et 5e rangée). Nous comprenons maintenant pourquoi un transfert avec 6 cm de longueur

foliaire, alors que la formation florale à 9° venait à peine de commencer, ne pouvait qu'être chose hasardeuse. On voit la même chose au tab. 2; ici les bulbes, ayant reçu une semaine 31°, ne furent plantés qu'après 6 semaines; 1e tab. 1 concerne des bulbes plantés immédiatement à 9°. Le tab. 3 donne l'évolution de ces fleurs sans la semaine à 31°, mais ayant reçu pendant 5 semaines de 23° à 31° et ensuite 9° ainsi que 7°. Les phases ont alors été déterminées 3 semaines après que les 6 cm de longueur foliaire eurent été atteints. Alors qu'avec 7°, la même longueur foliaire s'obtient 16 jours *plus tard* qu'avec 9°, le stade de formation florale est, 3 sem. plus tard, toujours plus avancé à 7° qu'à 9°. Avec 7° l'allongement des feuilles va donc plus lentement, la formation florale, probablement à peu près avec la même rapidité.

En ce qui concerne maintenant la réussite des fleurs, on peut prendre deux voies différentes.

I. Le résultat le plus précoce s'obtient en donnant une seule semaine à 31° et en plantant immédiatement après à 9°. Cependant, ce procédé exigeant beaucoup des bulbes, ceux-ci doivent avoir été bien nourris; les bulbes provenant des terres argileuses fournirent beaucoup plus de fleurs que ceux de terres sablonneuses; il convient de planter en terre sablonneuse riche. Les bulbes de terres sablonneuses qui avaient été quelques semaines dans les serres éclairés au néon donnèrent un nombre beaucoup plus grand de réussites. Pour ce traitement de forçage (1 sem. 31° etc.), il est recommandable de planter immédiatement; on ne laissera donc pas les oignons 6 semaines à sec dans le froid.

II. On donne d'abord 5 sem. 23° ou 25½° (et non 31°!); la floraison vient ainsi quelques semaines plus tard, mais beaucoup plus sûrement. On peut alors employer aussi bien des bulbes de sol sablonneux que de sol argileux, pourvu qu'ils soient robustes. Il est également permis de commencer par refroidir 6 semaines (jamais davantage!) à sec, du moins quand cela est souhaitable. Après cette température préalable, 7° est très recommandable et vraisemblablement plus favorable que 9°. Quand, à 7°, on a atteint une longueur foliaire de 6 cm, on peut déjà passer au 15° dans les serres; il est plus sûr de laisser encore ± 2 semaines à 7°, de sorte que la formation florale soit plus avancée. Ce point sera étudié plus minutieusement au cours de l'hiver 1938—1939. Dans un cas, nous avons eu de part et d'autre avec cette méthode (5 sem. à 23°, planter ensuite à 7°) 100 % de réussites dans deux années différentes. Si, lors d'une longueur foliaire de 6 cm, nous passions immédiatement de 7° à une température de 15°, la floraison se produirait du 16 au 20 février.

Nous devons attirer l'attention sur le fait que ces données furent acquises avec la variété Imperator; le traitement le plus approprié pour une variété comme le Wedgwood serait quelque peu différent.

Petrology. — *Leucite rocks of the active volcano Batoe Tara (Malay archipelago).* By H. A. BROUWER.

(Communicated at the meeting of December 17, 1938.)

The small volcanic island Batoe Tara or Komba is situated about 50 kilometers to the north of Lomblen, which is one of the Lesser Sunda islands east of Flores. It rises from the deep sea to a height of nearly 750 meters above sea-level. This interesting active volcano consists of different types of leucite rocks. From a period between 1849 and 1852 stronger volcanic action has been mentioned, but the present activity is limited to solfataric action. A description of the island has been given by HARTMANN [1]). We refer to his publication for a map and sketches of the volcano and for details about its morphology and the history of its activity.

We had an opportunity to visit the island during the afternoon of August 5th and the morning of August 6th 1937 when the government-steamer "Merel" on her way from the island of Wetar to Larantoeka in eastern Flores called at Batoe Tara and anchored in the bay on its western side. On all sides the volcano rises with steep slopes from the sea. Its form is not circular, especially on its eastern and western side the shore line bends inward. HARTMANN investigated already the part of the volcano near the crater and he mentions that the numerous lava flows nearly always come down to sea-level. We therefore investigated especially the lower part of the volcano, where wave action and subsidence have favoured the accessibility of good rock exposures. We had to follow the shore at low tide for investigations in the eastern half of the island. In attempting to land at the east coast our dinghy was smashed to pieces.

The rocks and their relative age.

Lava flows predominate and where the bare rock is exposed the lava is often seen to be cut by dykes of which the measured directions point to a more or less radial arrangement around the present center of eruption. Much of the products of the youngest eruptions must have come down in the gully on the eastern side of the volcano, where the crater wall is broken through. The slopes of the gully show bare older lavas, but exposures of the youngest volcanic products are found in the neighbourhood of the bay which forms the prolongation of the gully.

Bombs, agglomerates and tuffaceous rocks are found with the lava flows

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[1]) M. HARTMANN, Der Vulkan Batoe Tara. Zeitschr. f. Vulkanologie, **16**, 180—191 (1935).

of the volcano, but it seems that much of this material has descended along the slopes directly after their eruption or by later erosion and is now below sea-level.

The rocks of our collection can be divided into three groups.

1. lavas: leucitebasanites (partly transitions to leucitebasalts, leucitites and leucitetephrites).
2. dykes: leucitebasanites.
3. young biotiteleucitetephrites.

In the literature different names are used for the leucite rocks. In this paper the nomenclature of ROSENBUSCH is used. In the short description of nine rocks of Batoe Tara by HARTMANN (loc. cit.) a microscopical description is given of one of the rocks. As the composition of the groundmass of the other rocks is not known we cannot classify them exactly but they do not seem to differ essentially from the various rock types of which the characteristics are given below. As to their age HARTMANN only mentions of one of his samples that it was taken from a young lava flow. The rock is rich in olivine and therefore its mineralogical composition differs from the young volcanic products of our collection which are free from olivine. A short description of the various rock types will be given below.

Leucitebasanite lavas.

Because plagioclase and olivine have been found in all the rocks which were studied under the microscope they can be classified as leucitebasanites. Some rocks are poor in olivine and form a transition to the leucitetephrites and — if the plagioclase content decreases also — to the leucitites. Rocks with a high olivine- and a low plagioclase content form a transition to the leucitebasalts.

Leucite, plagioclase, augite, olivine and some iron ore occur as *phenocrysts*. In our collection large phenocrysts of *leucite* are seldom found, we found them up to nearly two centimeters in diameter in pebbles on the western shore of the island. In other rocks leucite is found in the groundmass only. The prevailing *plagioclase* of the phenocrysts is a basic bytownite with a narrow margin of more acid composition. Its quantity and size are varying. In some leucitebasanites only a few small plagioclase phenocrysts are found, they are entirely lacking in rocks with numerous phenocrysts of augite, olivine and varying quantities of leucite. The greenish *augite* often has a zonal structure with frequent alternations of zones with slightly different extinction angles. In some rocks it is the principal phenocryst with plagioclase but it is also found with much leucite or with much olivine. It is entirely lacking as phenocrysts in some rocks, which are poor in olivine and which contain larger crystals of plagioclase and numerous small crystals of leucite. The *olivine* occurs as phenocrysts and numerous small crystals are constituents of the groundmass, but in

some rocks which form a transition to the leucitetephrites it is found in small quantities only. Larger crystals of *iron ore* occur in most of the rocks, but they are much less abundant and smaller in size as compared with the other phenocrysts.

The *groundmass* of the leucitebasanites consists of the same minerals in varying quantities. The plagioclase is labradorite and andesine. Most of the rocks contain numerous small crystals of iron ore and also apatite. In some lavas a brownish glass is found between the other constituents of the groundmass. Biotite is rarely found in these rocks.

In the vesicular lavas the cavities are sometimes filled or partly filled with calcite or analcite or with both minerals together. In the latter case the analcite is found along the margin of the cavity, the calcite in its central part.

The chemical analyses of three lavas are given below:

	I	II	III
SiO_2	47.05	48.01	47.95
Al_2O_3	16.71	15.23	13.81
Fe_2O_3	4.50	3.19	3.96
FeO	4.64	5.50	4.76
MnO	0.13	0.17	0.16
MgO	6.01	7.18	8.21
CaO	12.23	11.75	12.19
Na_2O	1.83	1.97	1.65
K_2O	3.32	5.12	5.03
TiO_2	0.95	0.92	0.87
P_2O_5	0.56	0.84	1.00
H_2O+	0.79	0.29	0.48
H_2O-	0.63	0.09	0.15
CO_2	0.75	tr.	—
Cr_2O_3	0.01		
ZrO_2	tr.		
S	tr.		
SO_3	tr.	tr.	
SrO	0.01		
BaO	0.07		
	100.19	100.26	100.22

I. *Vesicular leucitebasanite.* Collected in the middle of the western slope of Batoe Tara about 20 meters above sea-level. Larger crystals consist of basic bytownite, leucite, augite, olivine and some iron ore. The groundmass contains the same minerals, the felspar is labradorite. The cavities are partly filled with calcite. Anal. E. F. BOUMAN.

II. *Vesicular leucitebasanite.* Collected to the south of the gully on the eastern side of Batoe Tara about 30 meters above sea-level. Larger

crystals consist of leucite and augite with less olivine and some smaller crystals of basic bytownite and iron ore. In the groundmass the felspar is labradorite and andesine, it contains also much leucite, augite and iron ore. Anal. E. F. BOUMAN.

III. *Leucitebasanite.* Collected at a rough estimate about 100 meters to the south of II and 25 meters above sea-level. Many larger crystals of augite, olivine and leucite and some small crystals of iron ore are found in a groundmass, consisting of the same minerals with labradorite and apatite. Anal. E. F. BOUMAN.

The differences in chemical composition of these lavas are not very important. The variations of the Al_2O_3 content are expressed in the varying amount of plagioclase and leucite, the MgO content increases with the amount of augite and olivine and the K_2O content with the amount of leucite.

Leucitebasanite dykes.

We collected dyke rocks in the western, northern and eastern part of the island. Most of the rocks are poor in larger phenocrysts. They are holocrystalline and two sizes of smaller crystals can often be distinguished. The larger sized of these crystals form the main mass of the rock and between them a fine grained mixture of the same minerals is found.

Their mineralogical composition does not differ essentially from that of the lava flows. Sometimes small quantities of biotite are found in the groundmass, showing that this mineral was stable during the final crystallization of the dykes. Resorption phenomena are lacking in this biotite.

Clear structural differences between the marginal and central parts of the dykes are found. A dyke with a thickness of 1.25 meters in the southern part of the east coast is holocrystalline in its central part, which consists of bytownite (with a narrow more acid margin), augite, leucite, olivine and iron ore. A fine grained groundmass is found between numerous crystals of somewhat larger size. The form of these larger crystals clearly shows that their margin crystallized simultaneously with the groundmass. In the marginal part of the dyke the groundmass is hypocrystalline with much of a dark opaque substance and the phenocrysts occur in well-developed crystals.

The chemical analyses of two dyke rocks are given in the table on the next page.

I. *Leucitebasanite* collected in the middle of the western slope of Batoe Tara, about 40 meters above sea-level. It contains numerous larger crystals of basic bytownite with a more acid margin, augite, olivine and iron ore in a groundmass with leucite, labradorite to andesine, augite, olivine, biotite iron ore and apatite. Anal. E. F. BOUMAN.

II. *Leucitebasanite,* central part of a dyke in the southern part of the

	I	II
SiO_2	48.71	45.38
Al_2O_3	18.21	18.51
Fe_2O_3	3.56	5.96
FeO	5.32	4.24
MnO	0.14	0.18
MgO	4.89	4.79
CaO	10.41	11.73
Na_2O	2.05	1.79
K_2O	4.00	3.81
TiO_2	0.95	1.17
P_2O_5	0.71	0.96
H_2O+	0.50	1.05
H_2O-	0.51	0.60
CO_2	—	—
Cr_2O_3	0.01	
ZrO_2	—	
S	0.06	
SO_3	—	
SrO	0.02	
BaO	0.13	
	100.18	100.17

east coast of Batoe Tara, containing plagioclase, augite, olivine, leucite, iron ore and apatite. Anal. E. F. BOUMAN.

There are no essential differences between the chemical composition of these dyke rocks and those of the lava flows. The higher Al_2O_3 content and the lower MgO content of the analysed dyke rocks can be explained by the decrease of olivine and augite with regard to plagioclase and leucite.

Young biotiteleucitetephrites.

The youngest volcanic products were collected south of the bay on the eastern side of the island. They contrast with the older rocks by the black colour of their surface. They have scoriaceous features and partly show stratification. The mineralogical and chemical composition of these biotiteleucitetephrites differs rather strongly from those of the older lavas and dykes. The *phenocrysts* are plagioclase, biotite, augite and some small crystals of leucite. There are also crystals of iron ore and apatite, which are larger in size than the constituents of the groundmass.

The plagioclase is found in zonal crystals principally consisting of bytownite and labradorite. The brown *biotite* partly shows resorption phenomena. Some of the smaller crystals are nearly entirely resorbed. The greenish *augite* sometimes has a zonal structure with alternation of zones

with slightly different extinction angles. The *leucite* is only found in some small crystals among the phenocrysts. The *groundmass* contains plagioclase (mostly andesine), leucite, iron ore, some augite and glass.

The chemical composition of one of these tephrites is given below:

SiO_2	53.11
Al_2O_3	19.07
Fe_2O_3	3.38
FeO	3.47
MnO	0.20
MgO	3.23
CaO	7.11
Na_2O	3.54
K_2O	5.25
TiO_2	0.65
P_2O_5	0.73
H_2O+	0.40
H_2O-	0.14
CO_2	—
SO_3	tr.
	100.28

Biotiteleucitetephrite, collected south of the bay, which forms the prolongation of the gully on the eastern side of Batoe Tara, about 20 meters above sea-level. It contains larger crystals of bytownite to labradorite, biotite, iron ore, apatite and some leucites in a groundmass containing andesine, leucite, iron ore, some augite and much glass. Anal. E. F. BOUMAN.

Differentiation in the Batoe Tara magma.

Most of the leucitebasanitelavas of our collection must be older than the dykes, which have been found cutting through them at many places. The differences in chemical composition of the analysed rocks are not important; the dykes have a lower MgO (and CaO) content and a higher Al_2O_3 content than the lavas. But more analyses are wanted for an exact judgment about the chemical differences which might exist between the two groups of rocks. The differences between the youngest volcanic products of our collection and the older lavas are more important, because there is not only a further increase of Al_2O_3 and a decrease of CaO and MgO but there is moreover an increase of SiO_2 and the alkalies, particularly of Na_2O.

The abundance of analcite in the cavities of some of the older vesicular lavas indicates that sodium was introduced after their consolidation.

Sodium has the tendency to escape from magmas into the surrounding rocks and it may be expected that it will concentrate in the higher parts of the magma during periods of quiescence of the volcano. This sodium will partly escape in the surrounding rocks and partly remains in the magma which comes to the surface at the time of a new eruption.

When a new eruption starts the material, which first comes to the surface may be enriched in sodium while the later products of the same period of activity may be the normal potash rocks. Rocks which were enriched in sodium in a similar way may also occur amongst the products of older eruptions, but we stated already that much of the fragmental material of the older eruptions seems to have descended along the steep slopes directly after their eruption or by later erosion and are now below sea-level.

Anatomy. — *Comparison of the endocranial casts of the Pithecanthropus erectus skull found by* DUBOIS *and* VON KOENIGSWALD*'s Pithecanthropus skull.* By C. U. ARIËNS KAPPERS and K. H. BOUMAN.

(Communicated at the meeting of December 17, 1938.)

At the meeting of November 27th, 1937 VON KOENIGSWALD published his first communication [1]) on the fossil fragment of a right mandible, which he ascribed to a Pithecanthropus specimen. The object was found in the Trinil layer of Central Java.

The R. ascendens of the mandible failed, the fragment being broken caudally behind the third molar, frontally between the 1st and 2nd incisivus. Of the teeth, the molars and the 2nd premolar were preserved. The shape of the arcus dentium was human in character, being medially concave. Although the alveolus of the 1st premolar was covered by a crust, it was evident that there was only one alveolus and not two as there are in apes.

The second premolar was typically pithecoid and could be ascribed to an ape if it were not that it closely resembles the Sinanthropus premolar, although its relief is somewhat clumsier.

The relief of the molars is more complex than in man, but less so than in Sinanthropus (as also the posterior premolar).

The length occupied by the three molars in the jaw (40.0 mm) exceeds the human molar length, coming very near that of the Orang-Utan molars (42.4 mm, GREGORY). In backward direction the size of the molars showed a considerable increase, while in recent man the opposite is seen, the third molar being reduced in man. Even in Sinanthropus and in the Heidelberg jaw the third molar is reduced. The above mentioned relation is only found in Simia (also in Dryopithecus) so that this relation is pithecoid according to VON KOENIGSWALD.

On the other hand the breadth of the third molar exceeds its length. It has a high L/Br. index, as only occurs in man.

The mandible itself differs from the anthropomorphous mandible by its shortness and roundness. The chin is oblique (fliehend), a spina interdigastrica (typical of anthropoids) fails. Frontally the mandible decreases in height. There is a strong insertion surface of the m. digastricus (fossa digastrica). There are considerable resemblances with the Heidelberg jaw and instead of one mental foramen there are three, which would be an exceptional thing for an anthropoid.

Concluding VON KOENIGSWALD states that, apart from the pithecoid features of the large second premolar, the unreduced third molar and the length occupied by the three molars, the character of the mandible is hominid. In the size of the premolar it comes nearest Sinanthropus, from which, however, the mandible itself differs by its increase in height in frontal direction in the molar region and its decrease in the chin region and by the lack of a torus mandibularis. Its general shape resembles more that of the Heidelberg jaw without being so massive.

Considering the great resemblance of the fragment with the jaw fragment

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[1]) G. H. R. VON KOENIGSWALD, Ein Unterkieferfragment des Pithecanthropus aus Trinilschichten Mittel-Javas. Proc. Kon. Akad. v. Wetensch., Amsterdam, **40**, 883 (1937).

of Kedoeng Broeboes described by DUBOIS, which fragment was stated by DUBOIS to be a Pithecanthropus fragment, VON KOENIGSWALD does not doubt that his mandible is of the same species, and — on account of the arguments mentioned above — he believes that its features are primitively hominid rather than anthropoid.

Since, however, already in the old Pleistocene of Java human relics occur (Homo modjokertensis VON KOENIGSWALD [1]), while in the young pleistocene layers of Ngandong OPPENOORTH [2]) discovered his Homo Soloensis (Javanthropus OPPENOORTH), VON KOENIGSWALD concludes that in the middle pleistocene, to which his Pithecanthropus belongs, Pithecanthropus already was a hominid relic.

At the meeting of January 29th, 1938 of the Academy of Sciences VON KOENIGSWALD gave a communication on a new Pithecanthropus skull found in the deepest layer of Trinil Java [3]).

After the reconstruction of the thirty pieces of which it consisted, the fragment appeared to have a striking resemblance with that of the Pithecanthropus skull found by DUBOIS. Both skulls show the same clumsy structure of the supra-orbital region, the same frontal flatness and small calotte height. Also the sudden bend in the occipital part of the skull is the same in both. Both skulls have a considerable postorbital narrowing and even the curious protuberance in the bregma region occurs in both. According to VON KOENIGSWALD no doubt both skulls are representatives of one species. This is confirmed by the measurements accompanying VON KOENIGSWALD's paper.

VON KOENIGSWALD considers his skull (and therefore also DUBOIS' Pithecanthropus) as being hominid, e.g. on account of the features of the temporal region, specially the deep mandibular fossa with a tuber mandibulare in front of it, as only found in Hominids (MARTIN and SCHWALBE). According to him this alone would justify calling it hominid [4]).

The meatus externus in VON KOENIGSWALD's skull lies under the elongation of the zygomatic bone, as also observed in man. VON KOENIGSWALD also believes that skull bones of such a thickness as found in this skull and in DUBOIS' Pithecanthropus do not occur in anthropoid apes, but should be considered as a fossil hominid feature.

On the other hand, by the absence of a mastoid process, present in man

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[1]) G. H. R. VON KOENIGSWALD, Erste Mitteilung über einen fossilen Hominiden aus dem Alt-Pleistocän Ost-Java's. Proc. Kon. Akad. v. Wetensch., Amsterdam, **39**, 1000 (1936).

[2]) OPPENOORTH, De vondst van paleolithische menschenschedels op Java. Mijningenieur, **6**, 106 (1932) and Ein neuer dilavialer Mensch von Java. Natur und Museum, **62**, 269 (1932).

[3]) G. H. R. VON KOENIGSWALD, Ein neuer Pithecanthropus Schädel. Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam, **41**, 185 (1938).

[4]) VON KOENIGSWALD is inclined to correlate the deepened fossa mandibularis of man with his faculty of speech.

and also in the Sinanthropus, Ngandong and Rhodesian skulls, his skull resembles that of anthropoids.

Considering the fact that both his and DUBOIS' Pithecanthropus skulls are adult but that the volume of his skull (750 cc) is smaller than that of DUBOIS' specimen (900—950 cc), VON KOENIGSWALD is inclined to accept that DUBOIS' skull is that of a male, his own of a female individual. He furthermore believes on account of the characteristics of the temporal region mentioned above and on account of the relation of frontal sinuses in DUBOIS' specimen (WEINERT) [1]) that Pithecanthropus was hominid. As another argument for this he refers to BLACK's and WEIDENREICH's opinion concerning the affinity of Pithecanthropus and Sinanthropus, the latter being doubtless hominid (capacity ♀ 1050 cc; ♂ 1100—1200 cc).

Nevertheless he emphasizes the primitiveness of this hominid as evidenced by the small skull capacity, the failing mastoid process, the occurrence of an unreduced third molar and the length of the molar insertion.

Fig. 1. Endocranial cast of the Pithecanthropus of VON KOENIGSWALD superposed upon the dorsal contour of the endocranial cast of Pithecanthropus erectus DUBOIS. Reduction ≠ $\frac{2}{3}$.

[1]) WEINERT, Neue Untersuchungen über die Calotte des Pithecanthropus erectus DUBOIS. Zeitschr. f. Ethnologie, 199 (1922). As WEINERT, however, states, frontal sinuses also occur in the Chimpanzee and Gorilla (cf. his Rassen der Menschheit, p. 9).

The statements of VON KOENIGSWALD have been criticized by DUBOIS. Referring for this criticism to DUBOIS' articles in our Proceedings of January and March 1938 [1]), I only mention here that DUBOIS repeats that

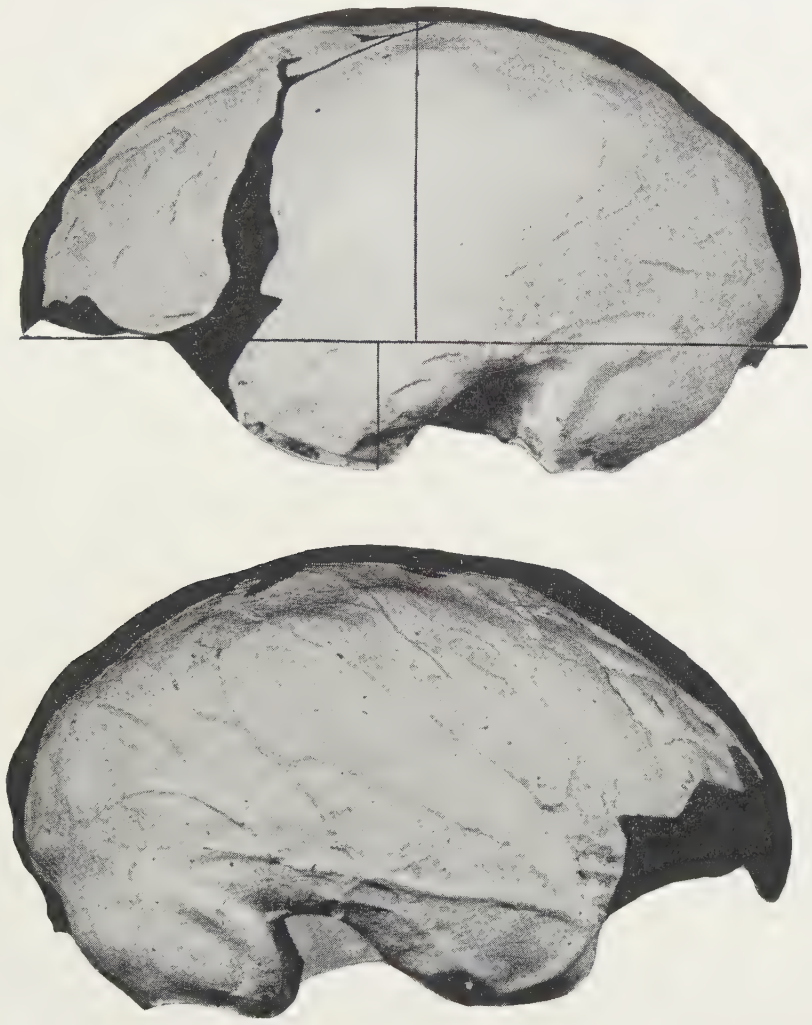

Fig. 2. The same as fig. 1. Lateral superposition. Reduction $\neq \frac{2}{3}$.

his Pithecanthropus skull was rather anthropomorphous (especially Hylobatid) in character and thus differs from Sinanthropus, which according to all authors is a primitive hominid, and furthermore that DUBOIS, granting the hominid features of VON KOENIGSWALD's mandible and skull, doubts their indentity with his Pithecanthropus fragments. He considers the skull

[1]) The mandible recently described and attributed to the Pithecanthropus by G. H. R. VON KOENIGSWALD, compared with the mandible of Pithecanthropus erectus described in 1924 by EUG. DUBOIS. Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam, **41**, 139 (1938) and The same: On the fossil human skull recently described and attributed to Pithecanthropus by G. H. R. VON KOENIGSWALD. Ibidem, p. 380.

TABLE I.

Measurements and Indices	Pithecanthropus erectus DUBOIS	Pithecanthropus erectus v. KOENIGSWALD	Sinanthropus pek. (E) Black	Javanthropus Ngandong V OPPENOORTH	Rhodesian
Capacity	900—950 cc ♂	750 cc ♀	1050 cc ♀ (11—1200 ♂)	1300 cc	1260 cc
Greatest length measured on our endocranial cast	15.39 cm	14.7 cm	15.99 cm	17.8 cm	17.35 cm
Greatest breadth measured on our endocranial cast	12.61 cm	12.00 cm	12.30 cm	13.72 cm	13.68 cm
L/Br. index calculated from these measurements	81.9 [1]	81.6	76.95	77.1	78.8
Max. height perp. on lat. hor. [2] / total length on lat. hor.	0.44 (R)	0.43 (L)	0.46 (L)	0.46 (L)	0.50 (L)
Dist. height perp. front. pole / Dist. height perp. occ. pole	1.11	1.01	1.19	1.23	1.32 (L)
Temp, occ. length [2] on lat. hor. / total length on lat. hor.	0.742 (R) ?	0.740 (L)	0.72 (L)	0.734?	0.76 (L)
Temp. depth [2] / total length on lat. hor.	?	0.167 (L)	0.149 (L) [3]	0.135	0.128 (L)
Temp. depth [2] / temp. occ. length on lat. hor.	?	0.225	0.208	0.184?	0.168 (L

to be of an immature individual, the mandible to be of a mature individual related to Sinanthropus and especially to OPPENOORTH's Homo Soloensis (Javanthropus).

By the kindness of Dr. VON KOENIGSWALD, Prof. K. H. BOUMAN and I each of us received an endocranial cast of VON KOENIGSWALD's Pithecanthropus, which we examined independently. For BOUMAN's description I refer to the Acta Neerlandica morphologiae normalis et pathologicae (Vol. II, 1938, p. 1). Here I wish to emphasize, as also Prof. BOUMAN did, the close resemblance of VON KOENIGSWALD's and DUBOIS' endocranial casts. Figs. 1 and 2 give the general contours of both casts, fig. 3 shows their fissural pattern. In the accompanying table I give some measurements and

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[1]) In his paper of Jan. 29th, 1938, p. 187 VON KOENIGSWALD gives 77.65. This, however, is a mistake, considering the length and breadth figures mentioned by him: $\frac{1545}{1260}$ being 81.5.

[2]) These calculations are made after the perpendiculars and distances on the lateral horizontal (passing underneath the tub. orbitale and occipitale pole) on the photos.

[3]) In my paper on Sinanthropus of Nov. 25th 1933 stands 0.173. This figure is wrong, not being calculated from the photo of the left hemisphere but from the drawing of the right.

indices of both endocranial casts from which it appears that the differences between the two Pithecanthropi are strikingly small.

Our table also shows that there is a gradual increase in the relative height from Pithecanthropus (average 0.43½) via Sinanthropus and Javanthropus (0.46) to the Rhodesian (0.50) and that the position of the height perpendicular, as indicated by the relation of its frontal and occipital distances (Pithecanthropus average: 1.06; Sinanthropus: 1.19; Javanthropus: 1.23; Rhodesian: 1.32) shifts more and more backward. There is a gradual decrease of the relative temporal depth in Pithecanthropus, Sinanthropus, Javanthropus and the Rhodesian compared with the total and temporo-occipital length, while both lengths gradually increase. The fact that the relation between total and temporo-occipital length in the Rhodesian (0.76) surpasses this relation in Pithecanthropus (0.74), while in Sinanthropus it is smaller (0.72), shows that in the Rhodesian the increase of the total length is more due to a relative increase of the temporo-occipital length, in Sinanthropus to a stretching of the frontal lobes. This may perhaps be correlated with the mesencephaly of Sinanthropus (77) and the brachencephaly (81.8) of Pithecanthropus, the latter affecting specially the frontal lobes.

In fig. 3 the fissures impressed on the frontal lobes of both Pithecanthropus casts are drawn. They show a resemblance beyond expectation, — confirming at the same time that the frontal fissural pattern as given by one of us in 1929 [1]) of DUBOIS' specimen is typical of this species.

As far as concerns the interpretation of the numbers added to the fissures, we refer to our former paper(s) on this subject.

We also call the attention to the peculiar fact that the arborisation of the dural arteria meningea media on the right hemispheres of DUBOIS' and VON KOENIGSWALD's specimens show a resemblance rarely found in this greatly varying [2]) vascular system [3]).

Turning to the question of the pithecoid or hominid character of these endocranial casts of Pithecanthropus, I would remark that just as in the dental system (length of the molar insertions, unreduced third molar, features of the second premolar) and in the calotte (small volume, failing mastoid) pithecoid features occur, next to hominid features (shape of the arcus dentium and of the mandible itself, presence of the tuber mandibulare, position of the meatus auditorius externus underneath the continuation of the zygomatic bone, massive skullbones), so also the endocranial relations show pithecoid and hominid features.

As far as concerns the fissures, I already pointed out that the frontal fissuration in Pithecanthropus shows far more affinities with that of Chimpanzees than ever observed in man, even in Neanderthal man.

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[1]) The fissures on the frontal lobes of Pithecanthropus erectus DUBOIS etc. Proc. Kon. Akad. v. Wetensch., Amsterdam, **32**, 182 (1929). See also KAPPERS, The evolution of the nervous system in invertebrates, vertebrates and man. (Bohn, Haarlem, 1930).

[2]) Cf. GIUFFRIDA–RUGGERI, Ueber die endocranischen Furchen der Arteria meningea media beim Menschen. Zeitschr. f. Morph. und Anthrop., **15**, 401 (1913).

[3]) This arborisation has a hominid character as also emphasized by WEIDENREICH; see his paper. The Ramification of the middle meningeal artery in fossil hominids and its bearing upon phylogenetic problems. Paleontologia Sinica N.S.D. N°. 3, Peking (1938).

The frontal fissures on the left hemisphere of a Chimpanzee published in our 1929 paper differ chiefly from those on the left hemisphere of

Fig. 3.

DUBOIS' Pithecanthropus by being steeper, more frontally curved (Cf. fig. 3 and fig. 4), which could be partly explained by the more brachencephalic shape of the brain (index Chimpanzee 84.2 Pithecanthropus 81.6).

Fig. 4. Fissuration of the left frontal lobe of a Chimpanzee.

DUBOIS is inclined to consider his Pithecanthropus skull rather Hylobatid than Chimpanzoid. Since the presently living Hylobatids have an encephalic 1/br. index of about 80, this index, as also their skull vault come nearer those of Pithecanthropus than the Chimpanzee's. The recent Gibbons and consequently also their brains are very small in comparison with the other anthropomorphs and their brains. Their frontal fissuration is too poor to be successfully compared with Pithecanthropus. But if ever a Gibbon of greater size and analogous index has lived, its fissures might perhaps resemble even more those of Pithecanthropus in being frontally less curved than in the Chimpanzee.

A pithecoid feature of the endocranial cast of DUBOIS' Pithecanthropus is seen in the possible indication of a lunate sulcus on the right occipital lobe on the level of the lambda suture, while in recent man (probably even on the Neanderthal cast of Dusseldorf) this sulcus lies a good distance behind the lambda suture. After all, DUBOIS' nomenclature "Pithecanthropus" seems to be the best expression for the intermediate character of this species.

Furthermore we believe that, although there are similarities also between the endocranial casts of Pithecanthropus and Sinanthropus, they should not be considered as belonging to one species, the latter having more hominid characteristics than the Pithecanthropus, as pointed out for the endocranial cast in 1933 [1]).

Leaving the final decision concerning the degree of relationship between Pithecanthropus and Sinanthropus to those who are more competent to judge the skeletal details, we take this occasion to compare again the Pithecanthropus and Sinanthropus endocranial casts, referring to figs. 3 and 5 and table I. Although the fissural impressions, specially on the left hemisphere of Sinanthropus, are not as evident as they are in the Pithecanthropus specimens, we believe the drawings given here approach their relations as closely as possible. From this it appears that the frontal fissures in Sinanthropus may come closer to the Pithecanthropus fissures

[1]) The fissuration in the frontal lobe of Sinanthropus pekinensis BLACK, compared with the fissuration of Neanderthalmen. Proc. Kon. Akad. v. Wetensch., Amsterdam, **36**, 802 (1933).

than was first expected. In this respect I would call attention to some resemblances between the right frontal fissuration of Sinanthropus and the

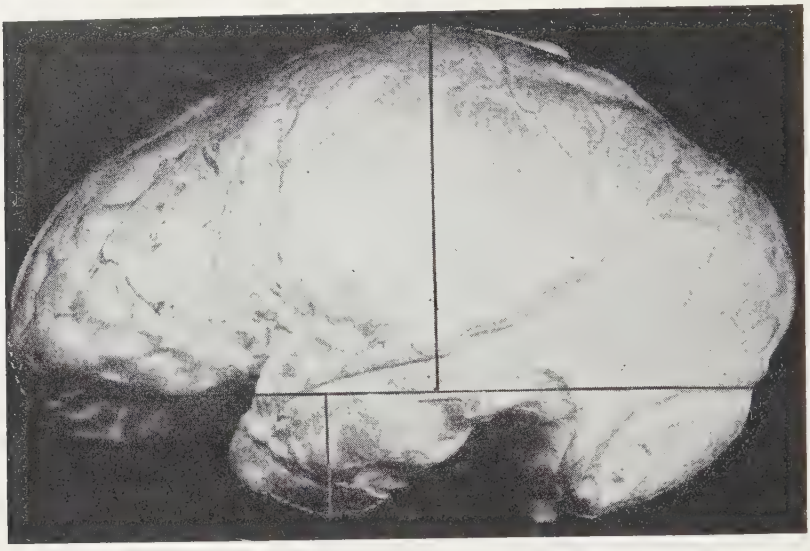

Fig. 5. Left hemisphere of the endocranial cast of Sinanthropus pekinensis (*E*) BLACK.

left frontal fissuration of Pithecanthropus DUBOIS. There also are resemblances between the left frontal lobes of Pithecanthropus VON KOENIGSWALD and Sinanthropus. Yet, in both Pithecanthropi the inferior frontal fissure [1]) (4) runs far less horizontally than in Sinanthropus, which thus shows a more hominid character, closely resembling the shape of the relation in the Rhodesian cast, as stated in our '33 paper. Also the separation of fissure 6 from 4 and its remaining attached to 7 is more hominid. In our description of the Sinanthropus cast we already called the attention to the possibility that *if* on the left hemisphere of Sinanthropus an indication of ELLIOT SMITH's lunate sulcus (or of the mesial continuation of this sulcus, the s. polaris superior) occurs, it lies behind the lambda suture (as it does in man), while in Pithecanthropus (right hemisphere) the impression that perhaps *might* be identified with it lies on the level of the lambda suture (a pithecoid relation).

From our table I it also appears that the general height index in Sinanthropus (0.46) is greater than in both Pithecanthropi 0.44 and 0.43), and that its greatest height perpendicular lies nearer the occipital pole, the relations between the frontal and occipital distance of this perpendicular on the lateral horizontal being 1.19 in Sinanthropus and 1.11 and 1.01 in

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[1]) I may call attention to the fact that this diagnosis of fissure 4 in both cases is confirmed by the course of CUNNINGHAM's external frontal artery, orbito-frontal artery (a. f. e) (BOUMAN and LEY's external orbito-frontal artery; POIRIER and SHARPY's artère de la 3ème circonvolution cérébrale), a pial artery.

the Pithecanthropi. Furthermore the relation of the temporal depth to the total as well as to the temporal length of the cast is smaller in Sinanthropus.

Also the volume (♀ 1050, ♂ 11—1200 cc) and general contours of the Sinanthropus endocranial cast are more human. They closely approach those of Homo soloensis, Javanthropus OPPENOORTH [1]), as stated elsewhere [2]) (Figs. 6*a* en 6*b*).

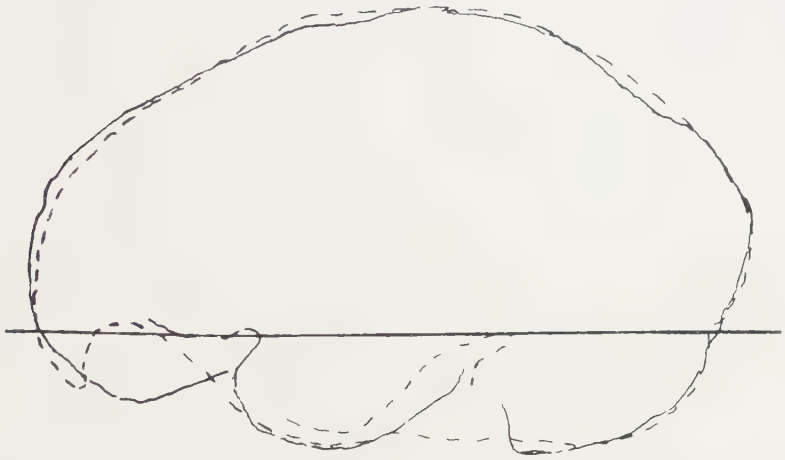

Fig. 6*a*. Continuous line left hemisphere Sinanthropus, dotted line left hemisphere Ngandong V endocranial cast, superposed with equal fronto-occipital length.

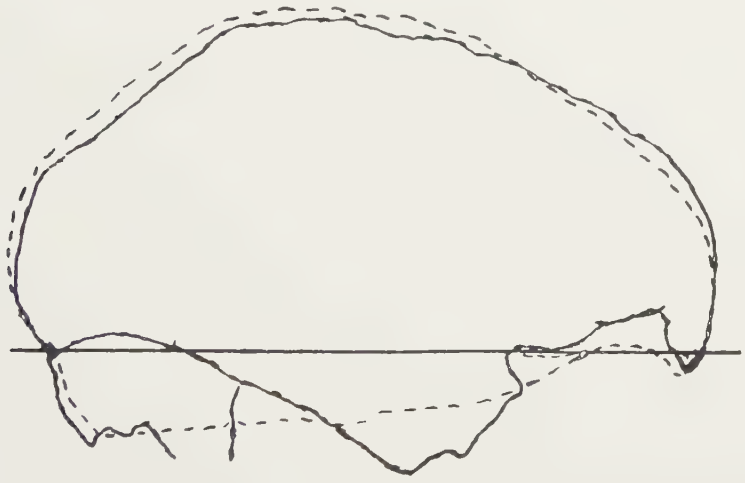

Fig. 6*b*. Continuous line right hemisphere Sinanthropus, dotted line right hemisphere Ngandong V endocranial cast, superposed with equal fronto-occipital length.

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[1]) OPPENOORTH, The place of Homo Soloensis among fossil men, in Early Man, ed. by Philadelphia, Lippencott (1937).

[2]) ARIËNS KAPPERS, The endocranial casts of the Ehringsdorf and Homo soloensis skulls. Journal of Anatomy, **71**, 61 (1936).

Also WEIDENREICH frequently emphasized the human character of Sinanthropus [1]). For the study of endocranial casts of fossil skulls in general, including hominid casts, I also call the attention to Dr. T. EDINGER's [2]) valuable papers.

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[1]) Cf. WEIDENREICH, Sinanthropus pekinensis. A distinct primitive Hominid. Proceed. of the Joint meeting of the anthropol. Society of Tokyo and the Japanese Society of Ethnology (1936). Ueber das phylogenetische Wachstum des Hominiden Gehirns. Kailbôgaku Zasshi, **9**, Aug. 1936; and Observations on the form and proportions of the endocranial casts of Sinanthropus pekinensis, other hominids and great apes. Paleontologia Sinica Ser. D, Vol. VII, Fasc. 4 (1936), and The relation of Sinanthropus pekinensis to Pithecanthropus, Javanthropus and Rhodesian Man. Journ. of the Roy. Anthr. Institute, Vol. **67**, 51 (1937).

[2]) T. EDINGER, Die Fossilen Gehirne. Springer, Berlin, 206 (1929). See also T. EDINGER, Paleoneurologie. Fortschritte der Paleontologie, **1**, 235 (1937).

Anatomy. — *Index curves of Asia and the Great Sunda islands.* By C. U. ARIËNS KAPPERS.

(Communicated at the meeting of December 17, 1938.)

Although the length breadth index of the head is by no means sufficient for a racial diagnosis, the typognostic value of this index, first emphasized by ANDERS RETZIUS, has been frequently confirmed, also in recent times.

So MORANT [1]) found that of 31 features of the skull six features have more value than the remaining 25 together, and that of these six the value of the length breadth index is about twice as influential for the coefficient of racial likeness as the other five.

More characteristic than the average index of a racial group are curves [2]) in which the individual indices are plotted and which show that the peak or peaks of the indices of a sufficiently large group usually coincide(s) with the peak or peaks of another group of the same race, even if their averages show some difference owing to the height of the peak(s) and the spread of the curve.

Such curves may also show that in addition to the peak(s) characteristic of a race a leap-like modification may occur in the form of an additional more brachycephalic peak, usually a fixation of an infantile stage. This fixation may have progressive value, as the development of the 78—80 peaks in a 73—75 index group when living under better circumstances, or a less favourable character, as the evolution of a hyperbrachycephalic peak in a brachycephalic people when living under poor circumstances.

This appears from BOAS' [3]) researches on Sicilian immigrants and his and GUTHE's [4]) on Jewish immigrants in America and from the curve of Armenian refugees measured by the KRISHNERS compared with the better situated Armenians measured by myself [5]). An analogous difference is observed between the head indices of the Palestinean and those of the poorer situated South-Arabian Arabs. It furthermore appeared from GUTHE's curves and from the curves I made of BOAS' material that the more brachycephalic peaks in the Jewish and Armenian curves again disappear in descendants living under better circumstances. That such additional peaks are fixations of an infantile stage is confirmed by KLEIN's researches [6]).

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[1]) MORANT, A preliminary classification of European races, Biometrika, **20** B, 301 (1928).

[2]) In all my curves the index figure 70 stands for 70—70.99 etc. If no other indication is given, the indices are those of adult males.

[3]) BOAS, Abstract of report on changes in bodily form of descendants of immigrants. Gov. Printing Office, Washington, D. C. (1911).

[4]) GUTHE, Notes on the cephalic indices of Russian Jews in Boston. Amer. Journal of Phys. Anthrop., **1** (1918).

[5]) ARIËNS KAPPERS, The degree of the changes in the cephalic index correlated with age and environment. Proc. Kon. Akad. v. Wetensch., Amsterdam, **38** (1935).

[6]) W. KLEIN, The degree of the developmental changes in the length breadth index of the head of Dutch Askenasim Jews. Proc. Kon. Akad. v. Wetensch., Amsterdam, **38**, 1021 (1935).

If the circumstances in which a race lives remain about the same, its index curve remains principally the same, however wide the spread of such a race may be. So the dark circumequatorial ulotrichians: the African negroes, Dravidians, Nesiots and the Pacific Melanoderms show the same index peaks and the same mutations: the 73—75 and 78—80 peaks, of which the former (73—75) may be considered the more primitive ones [1]).

As shown before (l.c.), the 73—75—78—80 cephalic curves (indicated already by the corresponding cranial peaks in the curves of paleolithic skulls) extend beyond the tropics with other races. So they are found along the coasts of the North Pacific and in the coastal regions and islands of the Arctic seas. In western direction the population living round the Mediterranean basin and along the coast of the Atlantic, up to Norway, shows these peaks. As this spread coincides with the spread of ancient conceptions of sun-religion, the spread of the svastica (FROBENIUS) and partly also with the spread of megalolithic monuments (which also prove the ethnic continuity of the Indians, Mediterraneans and early Atlantics [2]), including the real Nordic or Norwegian people), it seems to indicate the ways of migration of early humanity. Whatever other somatic changes were brought about by the different circumstances in these largely coastal regions, the head index of the people seems to be little affected by it.

Very different index curves from those of the circumequatorial, coastal Pacific, and Mediterrano-Atlantic races are found in the more central regions of Asia [3]).

The Paleo-Mongolids of S.W. Asia and Mongolians of Central Asia (Table I) show a successive change of the primitive index curve to a less primitive one.

The character of this change is not revealed by expressing the index of these groups by the average figure, but is made clear by the analytic method, by frequency curves, from which appears that in the so-called Paleo-Mongolid groups, next to the primitive index peaks a successive increase of the Mongolid 83 (and higher) indices occurs.

Naturally the question arises if this appearance and gradual increase of Mongolid indices is due to Mongolian admixture to originally primitive tribes or if it may be regarded as a gradual change of the index into a Mongolian one in individuals of an originally non-Mongolid race.

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[1]) The mutual relationship of the circumequatorial peoples and their derivatives is confirmed by the bloodtype and by ethnological facts, cf. ARIËNS KAPPERS, The character and spread of associated cephalic and cranial index peaks in Africa. Proc. Kon. Akad. v. Wetensch., Amsterdam, **39** (1936) and: The spread of primitive humanity and its links with the more differentiated races as revealed by cephalic and cranial index curves. I, Ibidem, **39** (1936) and II, Ibidem, **40** (1937).

[2]) With this name I indicate the people living in the European countries bordering on the Atlantic ocean.

[3]) See for the subject treated here also WOO and MORANT's paper: A preliminary classification of Asiatic races, bared on cranial measurements, in Biometrika, **24**, 108 (1932).

Evidently both may occur. This has been realized also by RISLEY, who in his "Peoples of India" speaks of Mongolo-Dravidian [1]) and Mongolid groups. So there seems no doubt that in some groups of Eastern Bengal a mixture has taken place between Mongol and Dravidian elements as in the Pamirs a mixture between Mongols and what might be Iranian groups. These mixtures are not always easy to disentangle. In most cases historical

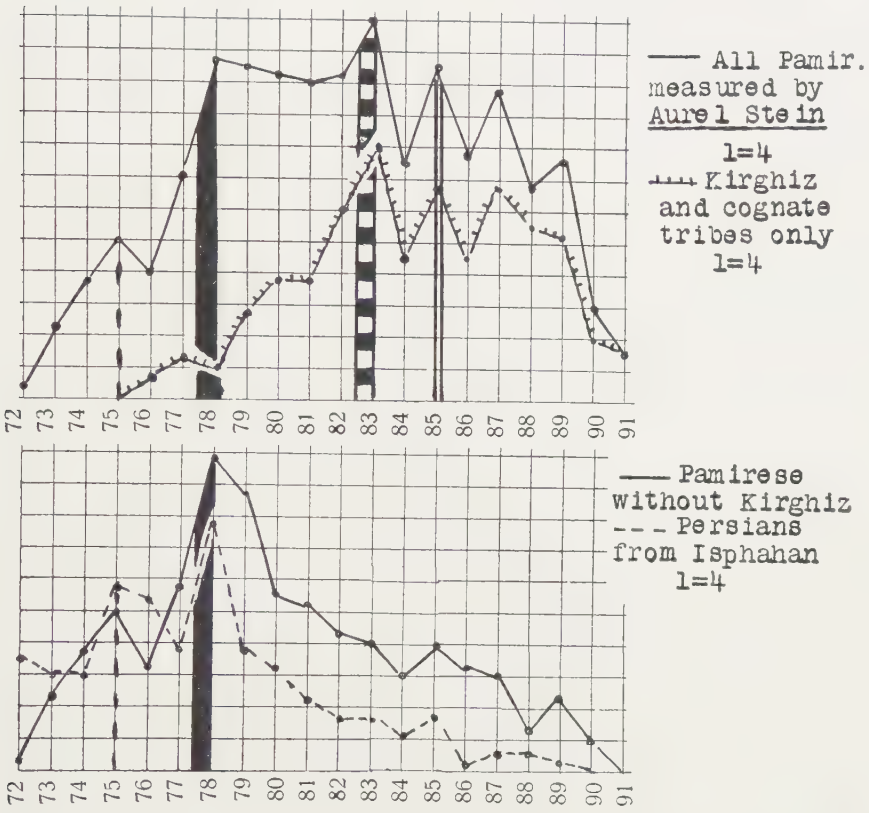
All Pamir. measured by Aurel Stein
1=4
Kirghiz and cognate tribes only
1=4
Pamirese without Kirghiz
Persians from Isphahan
1=4
72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91

Fig. 1.

or linguistic data have to help us. In some cases, however, the analysis of the index curve may give us valuable indications in so far as in a mixed group (cf. text fig. 1) the indices intermediate between those characteristic of each group show a considerable increase due to the addition of the feet (spread) of each index group. The intermediate part of the total curve then is more like a plateau, with less pronounced peaks. This is evident if

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[1]) As examples of such groups RISLEY mentions the Kayasaths and Brahman of Eastern Bengal measured by himself and the Kochh and Khasi measured by WADDELL, typical dolicho- and meso-cephalic tribes with Dravidian indices. As his Mongolids he mentions the Lepchas and Chakmas, the latter of which are very brachycephalic. Yet WADDELL believes that the Khasi and Kochh should already be considered as Mongolid. The Khasi are registered in Table I. fig. 1.

one makes a curve of all the Pamirese people measured by AUREL STEIN (published by JOYCE) amongst whom Iranian and Mongolian groups (Kirghiz) occur (see textfig. 1). Also the Koreans registered in Table I fig. 15 impress us as a mixture of a coastal pacific people with 78—80 peaks (like the Japanese) and a central Mongolian 83 peak group.

On the other hand there are groups or tribes which, as to their metrical and non-metrical facial features, impress us as Mongolid types (cheekbones, scanty hair, eyeshape etc.) and yet have very few typical Mongol indices (83 and higher). Probably such people are no mixture with real (brachycephalic) Mongols. Considering the fact that brachycephaly usually is dominant (FRETS), it most likely would also appear in the descendants of a mixture in whom Mongol facial features appear as a consequence of this mixture. Such groups in which the Mongol index is still rare, though in their facial characteristics Mongol features occur, may represent the primitive or Paleo-Mongolids, who stand still nearer the primitive tropical races than to the real Mongolians.

With some of these Paleo-Mongolid [1]) groups the primitive index type still is the only or nearly the only one. This appears from the curves of the Chinese Shan (or Shan ta Loke) measured by RISLEY, of WADDELL's Khasi of the Brahmaputra valley (Table I, fig. 1) and also from the curve of STEVENSON's Khams Tibetans (fig. 4). In the East-Himalayan Tibetans (fig. 4), the Burmese and Southern Chin (fig. 3) the 75—78—80 peaks are complicated with mongolid 83 and 85 elevations and with the Mois of Indo-China (fig. 2) also a 83 peak occurs.

In the Palaung (fig. 5), measured by VON EICKSTEDT and in RISLEY's Taung Thu and Talaing (figs. 6 and 7), the Mongolian peak (83) already is the highest although the primitive 78 and 80 peaks still are distinctly indicated.

Similarly with the Katchin and Karen (text fig. 2) a combination of the primitive 78 peak with an 81 and with the typical Mongol 83 (and 85) peak(s) occurs [2]).

With the Bashkirs (fig. 8), Samoyeds (fig. 9), Jenessei Ostyaks (fig. 10), Syrjans (fig. 11), Lapps, Kalmucks (fig. 12) and especially with the Hazara of Afghanistan [3]) (fig. 13) and Kirghiz (fig. 16) the primitive peaks disappear and the 83 and more brachycephalic peaks are outstanding. With the Kirghiz and cognate tribes (fig. 16) and with the Torguts, Telengetes (figs. 14, 19 and 20) and Buriates (fig. 17) the

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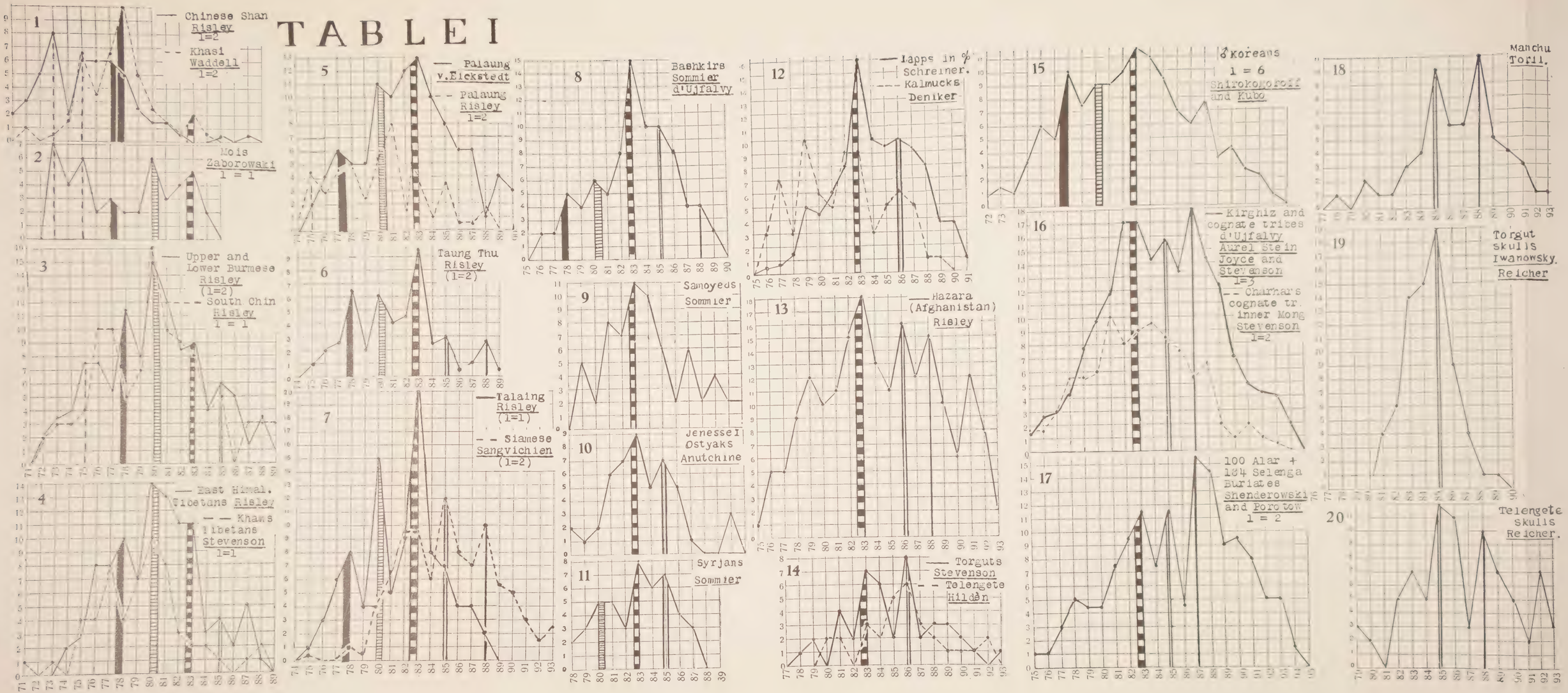
[1]) They are also called Palaungids after one of the tribes belonging to this group, cf. MENGHIN, Weltgeschichte der Steinzeit, Vienna, 1931.

[2]) As shown by the broken line in fig. 5, RISLEY's Palaung indices differ considerably from VON EICKSTEDT's. RISLEY's curve resembles the Katchin and Karen curves reproduced in textfigure 1, thus raising the question as to their belonging to the real Palaung.

[3]) According to their tradition the Hazara are a Mongol tribe left behind in Afghanistan by Djengis Khan. RISLEY, who classifies them with his Turko-Iranians, also states that they may be just as well Mongoloid.

TABLE I


1
Chinese Shan
Risley
1=2
Khasi
Waddell
1=2
2
Mois
Zaborowski
1 = 1
3
Upper and
Lower Burmese
Risley
(1=2)
South Chin
Risley
1 = 1
4
East Himal.
Tibetans Risley
Khams
Tibetans
Stevenson
1=1
5
Palaung
v.Eickstedt
Palaung
Risley
1=2
6
Taung Thu
Risley
(1=2)
7
Talaing
Risley
(1=1)
Siamese
Sangvichien
(1=2)
8
Bashkirs
Sommier
d'Ujfalvy
9
Samoyeds
Sommier
10
Jenessei
Ostyaks
Anutchine
11
Syrjans
Sommier
12
Lapps in %
Schreiner.
Kalmucks
Deniker
13
Hazara
(Afghanistan)
Risley
14
Torguts
Stevenson
Telengete
Hildén
15
3 Koreans
1 = 6
Shirokogoroff
and Kubo
16
Kirghiz and
cognate tribes
d'Ujfalvy
Aurel Stein
Joyce and
Stevenson
1=3
Charnay's
cognate tr.
inner Mong
Stevenson
1=2
17
100 Alar +
184 Selenga
Buriates
Shenderowski
and Porotow
1 = 2
18
Manchu
Torii.
19
Torgut
skulls
Iwanowsky.
Reicher
20
Telengete
skulls
Reicher.

more brachycephalic peaks are the highest. Finally with TORII's Manchu (fig. 18), the 85 and 87 peaks even are the only ones.

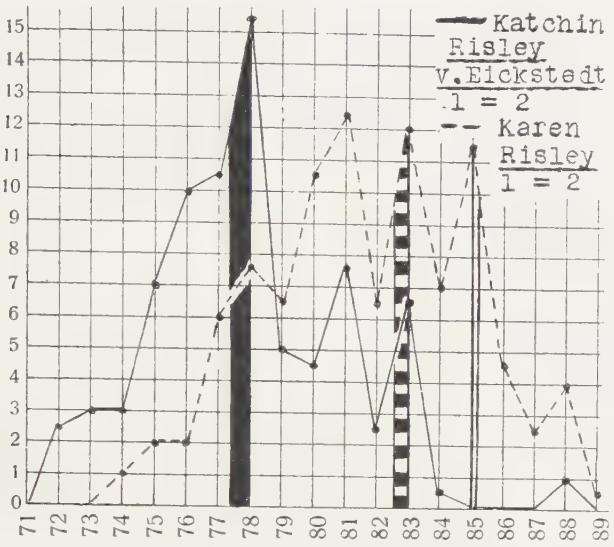


Fig. 2.

While with these tribes we find a gradual increase of brachycephaly, the index curves of the Arctic Sibirids (Table II): the Yakughirs (fig. 2), the Koryaks (fig. 3) are more primitive. With them the 78—80 peaks are again outstanding. With the Asiatic Eskimos (fig. 4) and the Kamtschadales (fig. 5; related to the Yakughirs), and even with the Lebedines and Tubalares (fig. 6), related to the Ob-Ostyaks (fig. 7) only one of the primitive index peaks (80) dominates. The Yakuts (Table II, fig. 1), usually considered also a Sibirid race, show a combination of the primitive 78—80 and the Mongol 83 and 85 peaks, pointing either to a transition of the Sibirid to a brachycephalic type, or, which is more likely, to a mixture of real Yakuts with central Mongols, such as the Buriates or Telengetes (cf. Table I), since the Yakuts penetrate far deeper into Central Asia than the other Sibirids, extending into the Buriate and Telengete areas [1]).

Table II, fig. 8 shows that the more primitive index type also occurs with the Tunguse and with the Japanese (fig. 11), who, according to MATSAMURA and TORII, are closely related to the Tunguse.

With the Japanese of South Noto (broken line fig. 11), measured by KOYA [2]), both the 78 and 80 peaks are distinct, the 78 one being the highest. In the other Hokuriku Japanese measured by this author and in

[1]) Considering the region where HECKER's Yakuts were measured, admixture with the Eastern Turki (which are also brachycephalic) is excluded.

The high 83 peak of the Chukchee (broken line, fig. I) may be due to Aleutian influence.

[2]) I am greatly indebted to Prof. TH. MOLLISON for sending me KOYA's important work: Rassenkunde der Ainu, published by the Japanische Gesellschaft zur Förderung der wissenschaftlichen Forschungen (Kanatawa, 1937) and his work on the Hokuriku Japanese.

MATSAMURA's Japanese the 80 peak prevails and brachycephalic indices are somewhat more numerous. Nevertheless the type of all these curves has the character of the coastal 78—80 curve.

The shape of the N. E. Chinese curve (fig. 9) though also showing a pronounced 78 peak, differs from the Japanese by the 81 index surpassing the 80 one. Besides, there is a slight 83 elevation in it probably indicating more Mongolian influence.

The fact that with SHIROKOGOROFF's Eastern Chinese the 81 peak practically is the only one, suggests that in KOGANEI's N.E. Chinese this peak may be chiefly due to the Eastern (Shantung) elements among his material.

The southern Chinese or Cantonese (Table II, fig. 10) again show the primitive 78 and 80 peaks, the former being the highest. The slight 82 elevation (indicated also in the Formosans) points to Malay admixture.

With the Formosans (fig. 12) the 80 peak, the only one with the Liu Kiu islanders, prevails. The large number of dolichocephalics among the Formosans is due to a primitive group amongst them, probably related to the Igorotes of Luzon (ALVAREZ [1]), MATSAMURA and MIYAUCHI [2])). I shall return to this in a following paper.

The Ainu of Hidaka, Iburi, Tokati and Sachalin measured by KOYA (fig. 13) similarly as those of Hokaida (Jezo) and Sachalin measured by KOGANEI, TORII and MONTANDON (broken line, fig. 13) give a still more primitive curve than the Formosans owing to the prevalence of 75 indices, complicated only with a high 78 and a much smaller 80 elevation. Brachycephalic indices practically fail with this people.

As stated by BÄLZ [3]), the Ainu are traced in Southern direction unto the Liu Kiu islands North of Formosa. According to KOGANEI on the adjacent continent no traces of this people are found. About their racial affinities there is a great deal of controversy (cf. VON EICKSTEDT's [4]) book). KOYA compares them with the Orotschones and Gilyaks but realizes himself that the high degree of brachycephaly of these Mongolian peoples (whose average index surpasses 85) is not in favour of a relationship.

Whereas several authors, amongst whom BÄLZ, HADDON, MONTANDON, consider them as a „Europid" remnant of North Western (European) origin, KOGANEI [5]) who has a great personal experience about the anthropology, history and prehistory of this people, links them up with Indian tribes. This author mentions especially the Todas but, as this people is far more dolichocephalic than the Ainu (cf. Table II, fig. 14), we would rather link them up with some other Indian tribes [6]) to which VON EICK-

[1]) ALVAREZ, Anthropos, Vol. **22**, 248 (1927).

[2]) MATSAMURA and MIYAUCHI, Proc. Imp. Ac. Tokyo, Vol. III, 576 (1927), Vol. IV, 408 (1928).

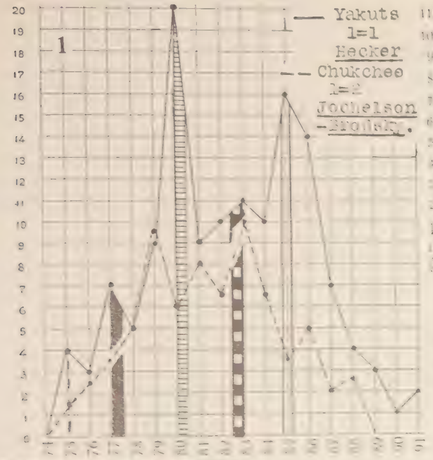
[3]) BÄLZ, Korr.bl. Deutsche Ges. f. Anthrop., Vol. **42**, 187 (1911).

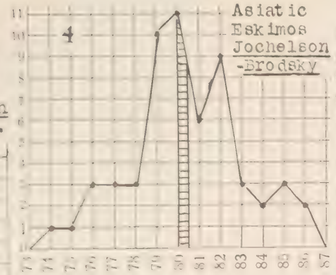
[4]) VON EICKSTEDT, Rassenkunde und Rassengeschichte der Menschheit, 1934.

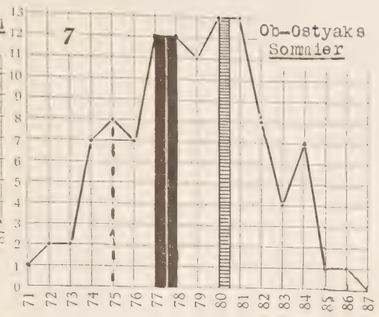
[5]) KOGANEI, Zur Frage der Abstammung der Ainu etc. Anth. Anz., **4** (1927).

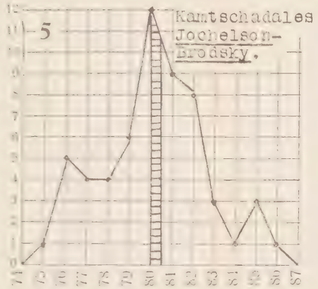
[6]) Some Dravidian groups, as the Mal Pahari (fig. 14) approach them in their indices, but stand nearer the Melanoderms.

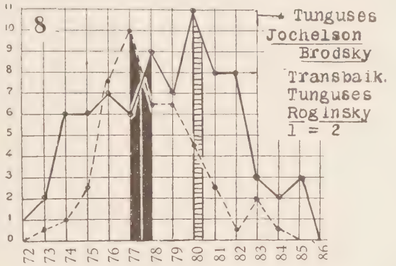
TABLE II

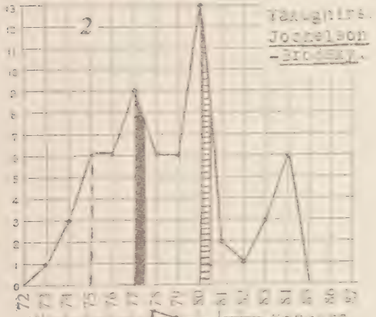

1
Yakuts
l=1
Hecker
Chukchee
l=2
Jochelson
-Brodsky.

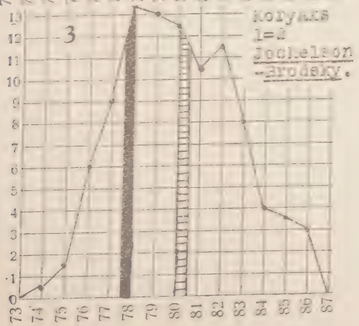

4
Asiatic
Eskimos
Jochelson
-Brodsky

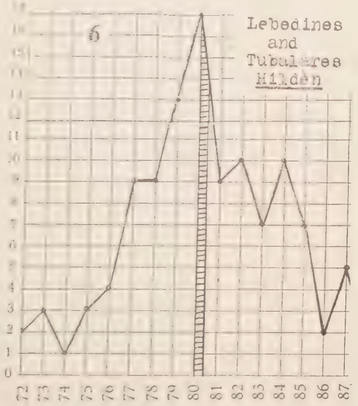

7
Ob-Ostyaks
Sommier

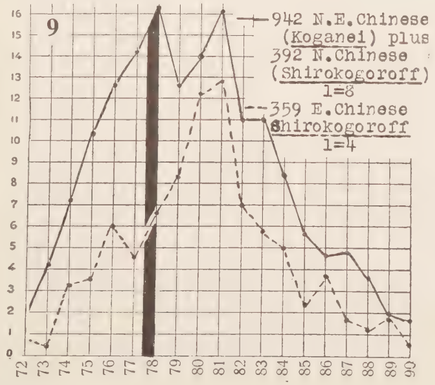

5
Kamtschadales
Jochelson-
Brodsky.

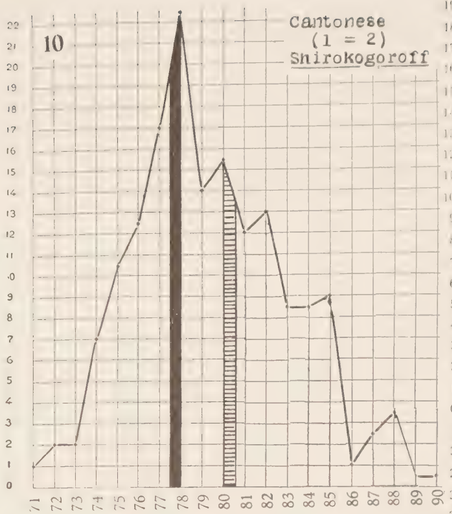

8
Tunguses
Jochelson
Brodsky
Transbaik.
Tunguses
Roginsky
1 = 2

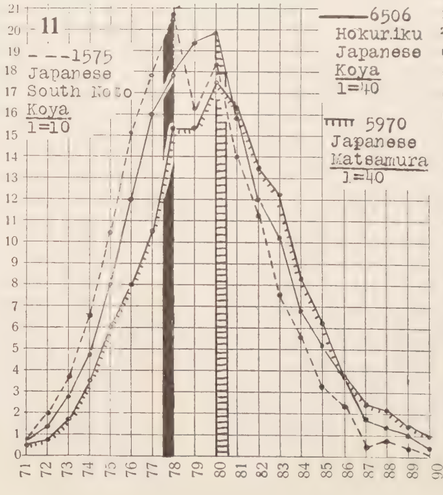

2
Yakutgirls.
Jochelson
-Brodsky.

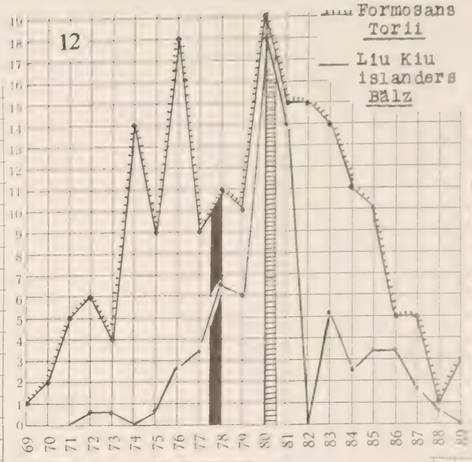

3
Koryaks
l=2
Jochelson
-Brodsky.

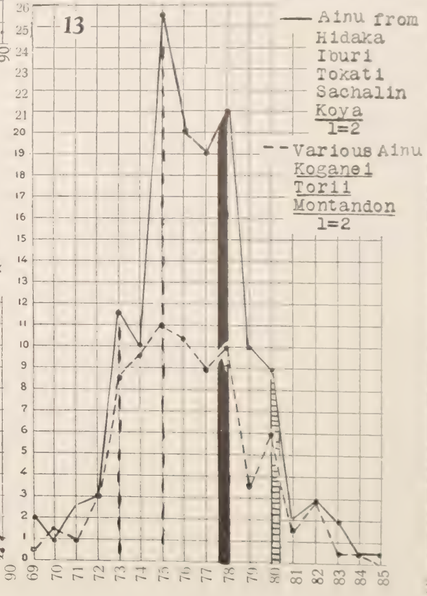

6
Lebedines
and
Tubulares
Hilden

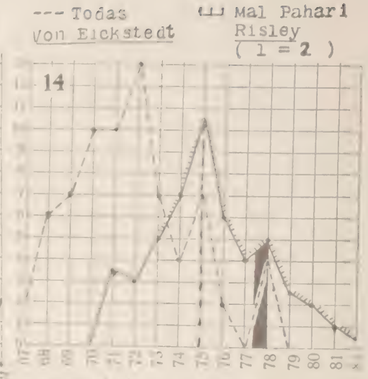

9
942 N.E.Chinese
(Koganei) plus
392 N.Chinese
(Shirokogoroff)
l=8
359 E.Chinese
Shirokogoroff
l=4

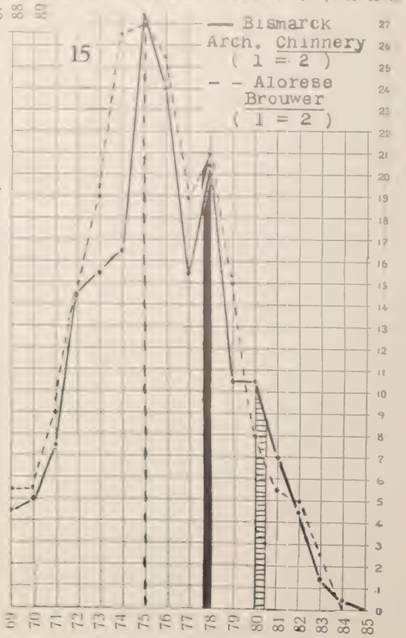

10
Cantonese
(1 = 2)
Shirokogoroff


11
1575
Japanese
South Noto
Koya
l=10
6506
Hokuriku
Japanese
Koya
l=40
5970
Japanese
Matsamura
l=40


12
Formosans
Torii
Liu Kiu
islanders
Bälz


13
Ainu from
Hidaka
Iburi
Tokati
Sachalin
Koya
l=2
Various Ainu
Koganei
Torii
Montandon
l=2


Todas
Von Eickstedt
Mal Pahari
Risley
(1 = 2)
14


15
Bismarck
Arch. Chinnery
(1 = 2)
Alorese
Brouwer
(1 = 2)

STEDT gave the name of Asiatic Europids. Features as found with the Ainu are frequent also with some Indian tribes (hence the name "Asiatic Europids", EICKSTEDT). Besides, the Ainu bloodtype is very different from any N.W. European and closely resembles some Indian bloodtypes [1]).

Index curves analogous to those of the Ainu are also found with Nesiots and Melanesians (Table II, fig. 15, and Table III, fig. 2) and the Ainu bloodtype, though nearer the Indian, also comes closer to the Nesiot and Melanesian type than to any European bloodtype. Considering the fact that some Indian tribes are closely related to the Nesiots and Melanesians, the similarity of the index curve and bloodtype of the Ainu with the Indian as well as with the Nesiot-Melanesian is not surprising.

The Great Sunda islands.

Turning to the Indian Archipelago (Table III), I may remind that in the Eastern part of the Archipelago: the Moluccas, Ceram, Flores, Timor, Sumba, Sumbawa, Pantar and Alor, the primitive Nesiot index type, resembling the Melanesian, is still prevailing, as appears from the Nesiot curves published in my preceding papers and from fig. 15, Table II of the present paper. With them the 75 (78 and 80) peaks are outstanding as they are with Melanesians.

These primitive peaks become rarer and rarer with the tribes in the Western part of the Archipelago.

In North Sumatra Paleo-Mongolid transitions (and admixtures) occur, as evidenced by the combination of the primitive 78—80 peaks with an outstanding 83 elevation. In the Atchinese curve (Table III, fig. 1) a high 83 peak occurs. For comparison with Paleo-Mongolids from the mainland I added (in the broken curve of fig. 1) the indices of the Taung Thu and Talaing from Burma. The similarity of this curve with the Atchinese one is striking.

The same is seen with KLEIWEG DE ZWAAN's Nias people (fig. 2), except his South Nias group. A similar contrast is observed among the Batta groups. Whereas the Karo Batta (continuous line fig. 3) and the related Simbiring (broken line fig. 3) measured by Miss KEERS, show rather primitive indices, the "other" Batta, registered in the coombed line of the same figure (especially the Pardembanem Batta measured by BARTLETT [2])), show considerable 83 and 85 elevations, suggesting Paleo-Mongolid elements.

On Borneo specially the Iban Dyaks [3]) contain a great number of

[1]) Cf. Table III in my second paper on the spread of primitive humanity and its links with the more differentiated races as revealed by cephalic and cranial index curves. Proc. Kon. Akad. v. Wetensch., Amsterdam, **40** (1937).

[2]) BARTLETT, The problem of Negrito and Vedda elements in the population of Sumatra. Proc. of the fifth Pacific congress Victoria and Vancouver, p. 2851 (1934).

[3]) According to BALNER and LEBZELTER's data and their analysis by KOLLER also the Kenyah Dyaks, though prevailingly dolicho-mesocephalic, contain a Mongoloid element (c). Anthropos, **30**, 505 (1935).

83 indices, which gives the continuous curve in fig. 4 a Paleo-Mongolid character, not caused by the prevailingly mesocephalic Klemantan Dyaks (registered separately in the broken curve of the same figure).

On Celebes the Paleo-Mongolid index type is indicated by the combination of the 77—80 peaks with a pronounced 83 and 85 elevation in the curve of the Bughi and Macassars (Table III, fig. 5), who apparently have Dyak relationships (for other Celeban groups, see p. 11).

While thus in the North-Western part of the Archipelago a Paleo-Mongolid index type may be traced, the population of the S. W. corner of the Asiatic mainland, the middle and Southern part of Sumatra and Java show different types of index curves (see Table III, fig. 6—14).

The curves of the Malacca Malays (fig. 6), Menang Kabau (fig. 7) and Djambians (fig. 10) of Sumatra and Javanese (fig. 12) show such a striking similarity to those of Cambodgians, Annamites, Tonkinese and Cochin-Chinese (fig. 9) that this confirms the relationship between these groups, which differ from the Paleo-Mongolids as well as from the real Mongols. With all these groups, and also with the Hakka measured by HAGEN (fig. 8), 82 (and 84) peak(s) dominate(s), combined (in figs. 6, 7, 8 and 10) with a small 78 elevation.

With the Siamese tribes from the S. W. corner of the Asiatic continent (figs. 11 and 13) as well as with the Javanese, Madurese and Sundanese (figs. 12 and 14) a successive increase of brachycephaly occurs. While the Cambodgians, Annamites, Tonkinese and Cochin-Chinese, measured by the French anthropologists (fig. 9) resemble the Malays from Malacca (fig. 6), the Djambians (fig. 10) and Menang Kabau (fig. 7) of Sumatra in having a high 82 (and smaller 84) elevation, the Khmer from Korat measured by CONGDON (fig. 11) show an additional 86 elevation which is still more pronounced with CONGDON's Paknampua and other Tai from Siam and with his Siamese Lao (fig. 13). In the Tai and Lao also a high 88 top occurs and in the Lao even a 90 elevation appears [1]).

The same increase of brachycephaly is observed on comparing the Sumatra and Java tribes of this stock. Whereas the Djambians measured by KOCH (fig. 10) only show an 82 peak and KLEIWEG DE ZWAAN's Menang Kabau (fig. 7) have 82 and 84 peaks, the Madurese (fig. 12) have a high 86 elevation and the Sundanese (fig. 14) a very pronounced 88 peak, indicated also in the Javanese curve (fig. 12), in which the additional 88 elevation, however, may be due to some West-Javanese (Sundanese) elements, the majority of the Javanese indices resembling those of the Menang Kabau people [2]).

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[1]) It is not astonishing that in addition to such groups giving curves of the Malay type there are Siamese groups which give a Paleo-Mongolid curve, as is observed with the indices added to Table I, fig. 7 (broken line), kindly given to me by Dr. SANGVICHIEN of the Department of Anatomy of the Siriraj Hospital, Bangkok.

[2]) A curve of male (perhaps prevailing East-) Javanese skulls also shows a primitive

Besides these more brachycephalized Malayan groups no more show the primitive index peaks (78 or 80) still indicated with the Malays from Perak and Malacca, the Menang Kabau, the Djambians, HAGEN's Hakkas and CONGDON's Khmer from Korat.

From this it appears that in the Malayan race we may distinguish more primitive (less brachycephalized) from less primitive (more brachycephalized) groups.

The Malays from Malacca, the Djambians, and Menang Kabau, HAGEN's Hakka having a 78 peak in addition to their 82—84 ones, might be called Proto [1]) or rather *Paleo-Malays*, as in their index they are more primitive than the other Malays. On the other hand the Lao and Tai from Siam, the Madurese, Sundanese and also those Javanese, who lost the primitive components and even acquired an 86 or (and) 88 elevation, might be called Deutero- or better *real Malays*, being more fully differentiated in the same direction. The Khmer from Korat apparently stand in between these two main groups.

Considering the fact that the data for these various groups are recorded by different anthropologists, the analogies in their index curves strike us the more and suggest that the Malays are a group of their own.

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76 Cranial index elevation, corresponding with the cephalic 78 Menang Kabau peak. Still this cranial index curve cannot be considered either as Paleo-Mongolid, as appears from a comparison with a Burmese skull curve.

[1]) The name Proto-Malays is used with different meanings (cf. KLEIWEG DE ZWAAN Science in the Netherlands East Indies, and Ons mooi Indië, p. 145 ff.). It is mostly used for people more primitive than the Javanese, Madurese and Menang Kabau Malays, populating the archipelago before the latter. Calling them also Pre-Malays, under this name very different tribes are grouped, some of which have nothing in common with the real Malays, e.g. the Ulu Aja of Borneo, the Igorots Batta, Dyaks, Sakai and other primitive dolichomesocephalic groups also called Indonesians. Used in this sense, the name Proto- or Pre-Malays suggests that it is from these peoples that the real Malays arose, which is by no means sure. It is better therefore not to use the name Pre- or Proto-Malays for these groups. KEANE (Ethnology, p. 328) also wishes to avoid any connection of the name Malay with such groups. He and HADDON (Races of Man, p. 32), however, call all the Oceanic Mongols Malays.

In QUIGGIN's and HADDON's revision of KEANE's Man Past and Present (p. 22) the name Pre-Malays is used again (now for the dolichocephalic element in Indonesia), while the name "Proto-Malays" is used for all the Oceanic Mongols. These authors, however, classify very different groups under the name Proto-Malays, as Batta, Sundanese, Javanese, Nicobars, Formosans, which cannot be classified together. Introducing the name Paleo-Malays, I refer to those Malay groups which, though having still partly primitive characters (also in their index peaks), at the same time show Malayan features (also in their indices), similarly as our Paleo-Mongolids show the characteristics of primitive groups together with characteristics of Mongol groups. With real Malays (most Siamese, Javanese and Sundanese) the Malayan features prevail to such an extent (also in their indices) that they should be considered as being fully "Malayanised". Although I agree with other authors that the Paleo-Malays in many ways resemble the Paleo-Mongolids, it seems possible and advisable to distinguish them as two separate and different branches of a common stock.

However great my admiration is for VON EICKSTEDT's magisterial book, I do not agree with this excellent anthropologist in classifying (p. 209) the Javanese with the Paleo-Mongolids. The Malays form a separate stock. Besides, the Javanese, Sundanese and Madurese lack those primitive index peaks whose combination with the typical Mongol peaks (83 and 85) is characteristic of Paleo-Mongolid curves.

VON EICKSTEDT himself gives as typical examples of the Paleo-Mongolid the Palaung and Katchin (Table I, fig. 5 and text figure 1). On comparison of the index curves of these peoples with our Javanese curve in Table III the difference is evident and the same holds good for the other Paleo-Mongolid groups registered in Table I (figs. 1—7), and Table III (figs. 1—5).

Considering the linguistic (and cultural: HEINE–GELDERN [1]) affinities between the Malays and Polynesians, it is interesting that the Javanese index curves so closely approach the Hawaian index curves, published in

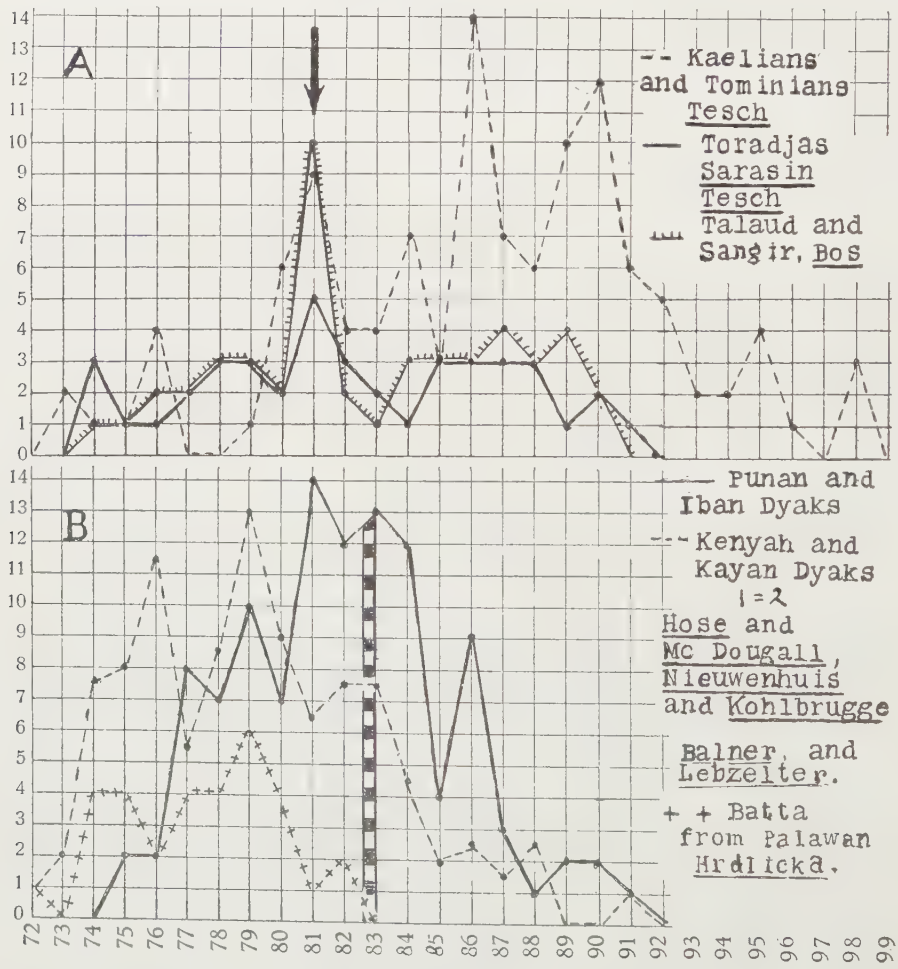
A
B
-- Kaelians and Tominians Tesch
— Toradjas Sarasin Tesch
Talaud and Sangir, Bos
— Punan and Iban Dyaks
-·- Kenyah and Kayan Dyaks
Hose and Mc Dougall, Nieuwenhuis and Kohlbrugge
Balner, and Lebzelter.
+ + Batta from Palawan Hrdlicka.

Fig. 3.

[1]) HEINE–GELDERN Urheimat und früheste Wanderungen der Austronesier, Anthropos, Vol. 27, 543 (1932).

my paper in these Proceedings, Vol. **40**, 1937 (Table I, figs. 10 and 11).

While in this paper most groups of the Archipelago, not belonging to the dolicho-mesocephalic stock (for which I refer to preceding papers) have been mentioned, other, especially dwarfish stocks will be dealt with in a following paper, dealing with Pygmy tribes and Veddoid peoples.

I only wish to add that some Celeban groups: the Toradja, Kailians and Tominians, measured by TESCH, apparently have Northern (Philippine) relationships, as already supposed by ADRIANI [1]), KRUYT [2]) and TESCH [3]).

This is confirmed by their index curves. So the Toradja curve (textfig. 3 A) shows a complete parallelism with that of the Sangir and Talaud islanders measured by BOS. In their bloodtype the latter [4]) resemble the Dusun Dyaks more than the Toradja. According to SARASIN the Dyaks are also related to the Toradja, especially the Kayan Dyaks described by NIEUWENHUIS (and KOHLBRUGGE). The Kayan (and Kenyah) Dyaks, however, are prevailingly mesocephalic, but the Iban and Punan Dyaks, though being more pronounced Paleo-Mongolid have an outstanding 81 index peak in common with the Toradja (textfig. 3 B). This peak also occurs with the Kailians (West-Toradja) and Tominians [5]) and the large number of hyperbrachycephalics amongst the latter probably results from the fact that the artificial fronto-occipital compression, practised also amongst the pure Toradja from Antja, is still more exaggerated by the Kailians and Tominians.

I hope that in giving the graphs in this and preceding papers I may have rendered some service to anthropologists, be it only by the fact that through the courtesy of several foreign and Dutch anthropologists I have been able to give several series of individual indices hitherto unpublished. As far as concerns the present paper I am especially indebted to Dr. P. STEVENSON, Peking, for the individual data of the central Asiatic groups measured by himself and those measured by SVEN HEDIN, to Dr. SANG-VICHIEN, Bangkok, for the Siamese measurements kindly taken for me, to Dr. E. D. CONGDON of the Long Island Medical School, Brooklyn (N. Y.), for his extensive and valuable series of indices of Tai, Lao and Khmer from

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1) ADRIANI, Overzicht van de talen van Midden-Celebes. Med. Ned. Zend. Gen., Vol. **42**, 1898; and ADRIANI en KRUYT, De Bare'e sprekende Toradjas van Midden-Celebes; Batavia 1912.

2) KRUYT, De West-Toradjas, Deel I, Hoofdst. 2. Verh. der Kon. Ned. Akad. v. Wetensch. Amsterdam, Afd. Letterkunde, Nieuwe reeks, Deel 40, 1938.

3) TESCH, Voorloopige aanteekeningen over anthropologisch onderzoek bij de bewoners van Midden-Celebes 1. Bloedgroepen. Geneesk. Tijdschrift voor Ned. Indië, Deel 78 (1938).

4) H. C. BOS, Bijdrage tot de anthropologie van de bevolking der Schouten eilanden. Diss. Amsterdam, '35, p. 143.

5) Although the Tominians are related to the Gorontalese (examined by LUBBERS, '93) their indices are about the same as those of the neighbouring Kailians.

Siam taken during his stay as Professor of anatomy at the Chulalongkorn University, Bangkok, and to Prof. E. VON EICKSTEDT for his series of Palaung, Katchin, and Toda indices.

I repeat my thanks to Miss KEERS for her measurements of the Karo Batta and Simbiring, to Dr. BOS for his data concerning the Talaud and Sangir islanders and to Dr. TESCH for his data concerning the Toradja, Kailians and Tominians of Celebes.

Palæontology. — Explanatory statement concerning the Communication of EUG. DUBOIS in Proceedings K. N. A. v. W., Vol. XLI, N°. 4, 1938: *On the fossil human skull recently described and attributed to Pithecanthropus erectus by* G. H. R. VON KOENIGSWALD.

In order to avoid a possible misunderstanding,

Prof. Dr. EUGÈNE DUBOIS declares

that on stating the, according to him, ascertained fact, that upon reconstruction of the skull described by Dr. VON KOENIGSWALD, which consists of numerous fragments, the original form has been modified, he by no means wants to say that this was done *on purpose*.

On the contrary, Prof. DUBOIS thinks that it was caused by the great technical difficulty in such a case to restore the original form by reconstruction. The inevitable minimal deviations in the junction of the edges of each time two fragments cause angularly considerable deviations of the opposite edges and these may be geometrically progressive.

This difficulty is so considerable that Prof. DUBOIS, who has a fifty years' experience of such work, declares not to be able in such a case to restore the original form.

Physics. — *The decay of the penetrating cosmic rays.* By E. M. BRUINS. (Communicated by Prof. J. CLAY.)

(Communicated at the meeting of December 17, 1938.)

§ 1. Recently it has been suggested that the penetrating component found by examination of the cosmic rays is produced by the positive and negative electrons falling from the cosmic space at a certain height in the atmosphere. The penetrating component itself would consist of heavy electrons which decay with a lifetime proportional to the energy. When they decay, ordinary electrons are formed which are absorbed according to the cascade theory. H. EULER and W. HEISENBERG [1]) estimated the lifetime of this heavy electron from absorption measurements and from the ratio of the number of penetrating and soft rays at great depths. However, some peculiarities, such as the dependence of the ratio of the soft and hard rays on the angle to the vertical at sea-level, are not given by their final formula. Below we shall discuss the modification of the energy-spectrum of the heavy electrons in the atmosphere.

§ 2. Let $m(h')$ be the mass per cm^2 per cm at the height h'. In the atmosphere may then be applied

$$m(h') = m_0 e^{-\beta h'}.$$

Suppose a is the loss of energie of a ray per cm per gram per cm^2 in electron-volt. The energyloss of a ray on proceeding along a path h, reckoned from the layer at a height H, where the penetrative rays are formed, is then

$$\triangle E = \int_0^h m_0 e^{-\beta(H-h)} a \, dh = A (e^{\beta h} - 1).$$

The magnitude A occurring here

$$A = \frac{m_0 e^{-\beta H} a}{\beta}$$

has the dimension of an energy and *is characteristic of the condition in the atmosphere*. For $H = 20$ KM, $a = 2 \times 10^6$ eV cm per gram, is $A = 1.64 \times 10^8$ eV. The constant of decay per cm of the path length is

[1]) H. EULER und W. HEISENBERG, Ergebnisse der exakten Naturw., **17**. H. EULER, Z. f. Physik., **110**, 692 (1938).

inversely proportional to the energy. To the number of penetrating rays N, formed with an energy E at a height H, consequently applies:

$$\frac{dN}{N} = -\frac{\alpha}{E - \triangle E}\, dh$$

from which for the number of particles with an energy $E—\triangle E$ at a distance h follows:

$$N = C(E)\left(\frac{e^{\beta h}}{E + A - A\, e^{\beta h}}\right)^{-\frac{\alpha}{\beta(E+A)}}.$$

If the energyspectrum of the rays at their formation is

$$N(E) = N_0\, E^{-s}$$

then follows from this

$$N = N_0\, E^{-s}\left[\frac{E + A - A\, e^{\beta h}}{E\, e^{\beta h}}\right]^{\frac{\alpha}{\beta(E+A)}}.$$

On measuring the energy in A, which is characteristic of the atmosphere, we find that

$$\underline{E = Ax}$$

$$N(x) = \overline{N}_0\, x^{-s}\left(\frac{x + 1 - e^{\beta h}}{x\, e^{\beta h}}\right)^{\frac{\alpha}{A\beta(x+1)}}.$$

Consequently, by absorption measurements only $\alpha/A\beta$ may be determined exclusively. The primary energy spectrum is multiplied by a factor which for high values of x approaches unity. For great energies, therefore, at every depth

$$N(x) \backsim \overline{N}_0\, x^{-s}.$$

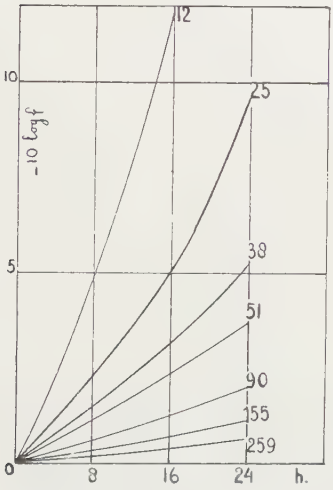

Fig. 1. Correction factor for the energyspectrum at $\tau = 10—6$ sec. at different heights for different energies. Lines for constant energy expressed in A.

A different choice of H, a, m_0, α, β only causes a change of the exponent of the correction factor. The magnitude $\alpha = \mu c^2/\tau c$ measures the proportion between the "rest-energy" and the "lifetime". We suppose $\mu c^2 = 8 \cdot 10^7$ eV. The correction may be obtained by means of fig. 1. *The energy spectrum shows a maximum which with increasing depth and diminishing lifetime is shifted towards a higher energy.* In the figures 2, 3 and 4 and the tables I, II, III and IV the energy spectrum is found for varying lifetime at depths of 8, 16 and 24 KM. The general behaviour of the energy spectrum might also be described as a change of the exponent s,

which holds good for great depths and runs via the value zero, reached in the maximum of the energy spectrum, to negative values. This change must then be found as well in the integral spectrum.

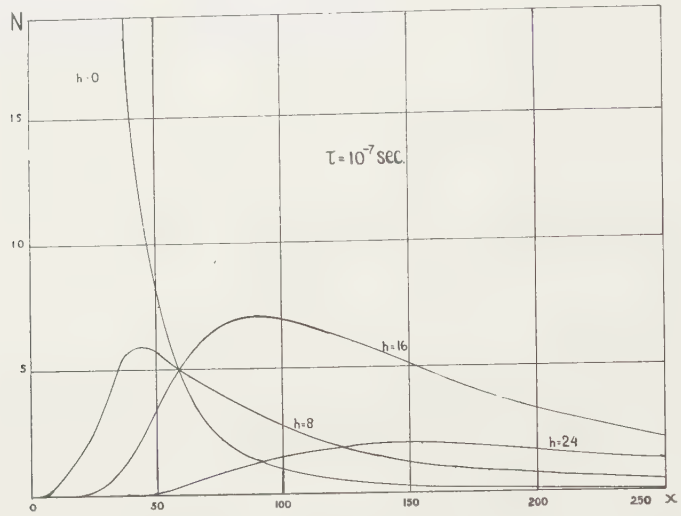

Fig. 2. Energyspectrum for $\tau = 10^{-7}$ sec. The given values have been multiplied for $h = 8$ KM by 10^7 and for $h = 16$ and 24 KM by 10^8.

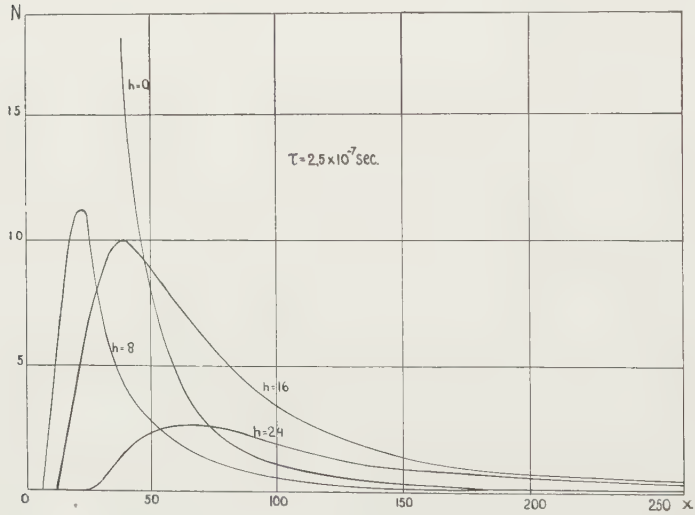

Fig. 3. Energyspectrum for $\tau = 2.5 \times 10^{-7}$ sec. The given values have been multiplied for $h = 8$ KM by 10^6 and for $h = 16$ and 24 KM by 10^7.

The total number of particles with an energy greater than qA is

$$N(q) = \overline{N}_0 \int_q^\infty x^{-s} \left(\frac{x + 1 - e^{\beta h}}{x\, e^{\beta h}} \right)^{\frac{\alpha}{A\beta(x+1)}} dx.$$

If we assume here that

$$xy = q \text{ and } s = 3$$

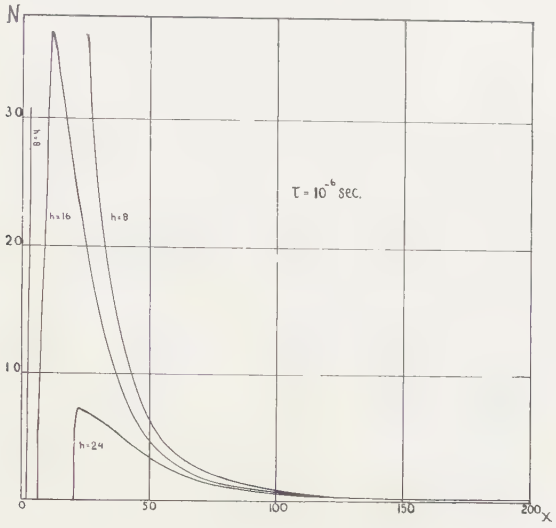

Fig. 4. Energyspectrum for $\tau = 10{-}6$ sec. The given values have been multiplied by 10^6.

then is

$$N(q) = \frac{\overline{N}_0}{q^2} \int_0^1 y \left(\frac{q - (e^{\beta h} - 1)\, y}{q\, e^{\beta h}} \right)^{\frac{\alpha y}{A \beta (y+q)}} dy = \frac{\overline{N}_0\, C(q)}{2\, q^2}.$$

TABLE I. $\tau = 10^{-7}$ sec.

x	$h=0$	$h=8$	$h=16$	$h=24$
5.5	6.011—3	6.89 —15	—	—
12	5.787—4	6.90 — 8	—	—
25	6.400—5	2.426— 7	6.433—10	1.45 —14
38	1.823—5	5.575— 7	1.256—8	8.09 —11
51	7.521—6	5.668— 7	3.624—8	1.292— 9
64	3.815—6	4.692— 7	5.655—8	4.658— 9
77	2.190—6	3.975— 7	6.761—8	9.180— 9
90	1.372—6	3.200— 7	7.093—8	13.435— 9
142	3.492—7	1.361— 7	5.436—8	20.02 — 9
155	2.685—7	1.156— 7	4 896—8	19.76 — 9
168	2.109—7	9.696— 8	4.395—8	19.13 — 9
181	1.686—7	8.197— 8	3.923—8	18.27 — 9
194	1.370—7	6.992— 8	3.532—8	17.65 — 9
259	5.756—8	3.480— 8	2.091—8	12.36 — 9

$E = Ax$ $\quad A = 1.64 \times 10^8$ eV $\quad h =$ depth in KM

The whole numbers indicate the exponent of 10 of the factor by which all given values must be multiplied.

TABLE II. $\tau = 2.5 \times 10^{-7}$ sec.

x	$h = 0$	$h = 8$	$h = 16$	$h = 24$
5.5	6.011—3	1.00 —7	—	—
10	1.00 —3	—	6 —10	—
11	7.514—4	—	2.99 —11	—
12	5.787—4	6.208—6	6.15 — 9	—
25	6.400—5	1.075—5	6.531— 7	8.88 —9
38	1.823—5	4.518—6	9.910— 7	1.317 —7
51	7.521—6	2.674—6	8.910— 7	2.343 —7
64	3.815—6	1.650—6	7.081— 7	2.608 —7
77	2.190—6	1.108—6	5.448— 7	2.451 —7
90	1.372—6	7.664—7	4.195— 7	2.155 —7
129	4.658—7	3.106—7	2.051— 7	1.200 —7
142	3.492—7	2.396—7	1.659— 7	1.1131—7
155	2.685—7	1.917—7	1.359— 7	9.456 —8
164	2.109—7	1.546—7	1.126— 7	8.074 —8
181	1.686—7	1.246—7	9.409— 8	6.932 —8
194	1.370—7	1.047—7	7.965— 8	5.988 —8
259	5.756—8	4.706—8	3.839— 8	3.111 —8

TABLE III. $\tau = 5.0 \times 10^{-7}$.

x	$h = 8$	$h = 16$	$h = 24$
5.5	2.459—5	—	—
10	—	7.968—7	—
11	—	14.99 —7	—
12	5.994—5	23.18 —6	—
25	2.098—5	6.465—6	7.538—7
38	9.075—6	4.251—6	15.50 —7
51	—	2.587—6	13.28 —7
64	3.525—6	1.644—6	12.55 —7
77	1.557—6	1.092—6	7.327—7
90	1.025—6	7.586—7	5.438—7
129	3.803—7	3.091—7	2.364—7
142	2.892—7	2.407—7	1.972—7
155	2.269—7	1.910—7	1.593—7
168	1.805—7	1.541—7	1.304—7
181	1.460—7	1.260—7	1.081—7
194	1.197—7	1.045—7	0.906—7
259	5.205—8	4.701—8	0.423—7

TABLE IV. $\tau = 10^{-6}$.

x	$h = 8$	$h = 16$	$h = 24$
5.5	3.844—4	—	—
10	—	2.823—5	—
11	—	3.336—5	—
12	1.862—4	3.662—5	—
25	3.665—5	2.034—5	6.946—6
38	1.286—5	8.852—6	5.315—6
51	5.807—6	4.411—6	3.160—6
64	3.094—6	2.504—6	1.950—6
77	1.847—6	1.547—6	1.267—6
90	1.186—6	1.020—6	0.864—6
129	4.209—7	3.794—7	0.332—6
142	3.178—7	2.899—7	0.249—6
155	2.468—7	2.265—7	0.207—6
168	1.951—7	1.803—7	0.166—6
181	1.568—7	1.457—7	0.135—6
194	1.280—7	1.196—7	0.111—6
259	5.473—8	5.202—8	0.050—6

The value $\frac{1}{2}C(q)$ of the integral measures the deviation of the integral spectrum which would be found without the occurrence of the spontaneous decay

$$N_0(q) = \frac{N_0}{2q^2}.$$

For high values of q the integrand approaches to y and consequently $C(q)$ to unity.

From measurements with absorbing material, where the influence of the decay may be neglected, the integral energyspectrum is to be found. By means of graphical integration we find the following values of C and τ for the total number of particles at $h = 24$ KM.

τ	10^{-5}	10^{-6}	5×10^{-7}	2.5×10^{-7}	10^{-7}
C	0.80	0.202	0.078	0.026	0.0076

Since further

$$\log N(q) = \log N_0(q) + \log C$$

we may obtain the lifetime τ from the size of the deviation from the

straight line in the *log N—log d* diagram ($d =$ thickness of the absorbing layer of water), which was found for great thickness by J. CLAY, A. VAN GEMERT and P. H. CLAY [1]). The deviation from the straight line occurring between 0 and 50 meters of water yields

$$\tau = 1{,}5 \times 10^{-6} \text{ sec.}$$

Amsterdam. *Natuurkundig Laboratorium.*

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[1]) J. CLAY, A. VAN GEMERT and P. H. CLAY, Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam, **41**, 694 (1938).

Mathematics. — *On the solutions of algebraic differential equations.* By K. MAHLER. (Communicated by Prof. J. G. VAN DER CORPUT).

(Communicated at the meeting of December 17, 1938).

Let

$$f(z) = a_0 + a_1 z + a_2 z^2 + \ldots \quad \ldots \ldots \quad (1)$$

be a convergent or divergent power series with coefficients in a finite algebraic field K, which formally satisfies an algebraic differential equation; i. e. there is a polynomial $F(z, y_0, y_1, \ldots, y_m) \not\equiv 0$ in K, such that identically in z [1])

$$F(z, f(z), f'(z), \ldots, f^{(m)}(z)) = 0. \quad \ldots \ldots \quad (2)$$

In his Groningen Thesis [2]), J. POPKEN proved the following

Theorem 1: *There is a positive number c independent of n, such that for all sufficiently large indices, either*

$$a_n = 0, \quad or \quad |a_n| \geqq exp\left(-c\, n\, (log\ n)^2\right).$$

The proof then given was rather complicated. In this note, I give a simpler proof, which depends on the following results of G. PÓLYA [3]):

Theorem 2: *There is an infinite sequence $a_0, a_1, a_2, \ldots$ of positive integers, such that all numbers $a_n\, a_n$ $(n = 0, 1, 2, \ldots)$ are algebraic integers, and such that*

$$\frac{log\ a_n}{n\ (log\ n)^2} = O(1).$$

Theorem 3: *There is a positive number c_1, which does not depend on n, such that for all sufficiently large indices n,*

$$|a_n| \leqq n!^{c_1}.$$

---

[1]) It suffices to suppose that the TAYLOR coefficients a_n are algebraic and that $f(z)$ satisfies an equation (2). For then, without loss of generality, the coefficients of the polynomial F may be assumed to be algebraic, and therefore the a's can be expressed as rational functions with rational coefficients in a finite number of the a's and in the coefficients of F.

[2]) Amsterdam 1935, N.V. Noord-Hollandsche Uitgeversmaatschappij, Satz 12.

[3]) C. R. **201** (1935), p. 444, first two theorems. I need these theorems only in the special case of rational coefficients a_n.

Proof of Theorem 1: Without loss of generality, the coefficients of the polynomial F may be assumed to be rational numbers [4]). Let the field K be of degree N, and θ be a generating number of this field; hence

$$a_n = \sum_{h=0}^{N-1} A_{hn}\,\theta^h \qquad (n = 0, 1, 2, \ldots), \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where the A_{hn} are rational numbers. Put

$$f_h(z) = \sum_{n=0}^{\infty} A_{hn}\,z^n \qquad (h = 0, 1, \ldots, N-1) \quad . \quad . \quad . \quad . \quad (4)$$

and for arbitrary t

$$f(z\,|\,t) = \sum_{h=0}^{N-1} t^h\,f_h(z), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

so that

$$f(z) = f(z\,,\theta). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Substituting in F, we get

$$F\left(z, f(z\,|\,t), \frac{\partial f(z\,|\,t)}{\partial z}, \ldots, \frac{\partial^m f(z\,|\,t)}{\partial z^m}\right) = \sum_{n=0}^{\infty} P_n(t)\,z^n, \quad . \quad . \quad (7)$$

where the $P_n(t)$ are polynomials in t with rational coefficients.

Suppose now that $\theta_0, \theta_1, \ldots, \theta_{N-1}$ are the N different conjugates of θ in the field of all complex numbers. Since by (2)

$$P_n(\theta) = 0 \qquad (n = 0, 1, 2, \ldots),$$

obviously also for $h = 0, 1, \ldots, N-1$

$$P_n(\theta_h) = 0 \qquad (n = 0, 1, 2, \ldots).$$

Therefore by (7), the N power series

$$f(z\,|\,\theta_h) \qquad (h = 0, 1, \ldots, N-1). \quad . \quad (8)$$

all satisfy algebraic differential equations, viz. the same equation (2).

Now it is easily shown that if $g_1(z), \ldots, g_r(z)$ are power series which satisfy algebraic differential equations, and if $\lambda_1, \ldots, \lambda_r$ are constants, then the series $\lambda_1 g_1(z) + \ldots + \lambda_r g_r(z)$ also satisfies a certain algebraic differential equation [5]). Therefore the N functions (4) must be solutions

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[4]) If necessary, multiply F by its conjugate polynomials with respect to K.

[5]) Put $\lambda_1 g_1(z) + \ldots + \lambda_r g_r(z) = g(z)$ and suppose that $F_1 = 0, \ldots, F_r = 0$ are the differential equations for $g_1(z), \ldots, g_r(z)$. By differentiating these equations a sufficient number of times and by considering $g(z)$ as known, we obtain so many equations for the functions $g_1(z), \ldots, g_r(z)$ and their differential coefficients, that we can eliminate them.

of algebraic differential equations; for by (5) they can be expressed linearly with constant coefficients by means of the functions (8).

Write the rational numbers A_{hn} as

$$A_{hn} = \frac{P_{hn}}{Q_{hn}} \qquad (h = 0, 1, \ldots, N-1 ; n = 0, 1, 2, \ldots), \quad . \quad (9)$$

where P_{hn} and $Q_{hn} \geqq 1$ are relatively prime integers. By the Theorems 2 and 3, there is a positive constant c_2, such that for $h = 0, 1, \ldots, N-1$ and for all sufficiently large n

$$max\,(|P_{hn}|, |Q_{hn}|) \leqq exp\,(c_2\, n\, (log\, n)^2). \quad . \quad . \quad . \quad . \quad (10)$$

Put

$$q_n = \prod_{h=0}^{N-1} Q_{hn}, \quad p_{hn} = A_{hn}\, q_n \qquad (h = 0, 1, \ldots, N-1 ; n = 0, 1, 2, \ldots), \quad (11)$$

such that all p_{hn} and $q_n \geqq 1$ are integers, and that

$$A_{hn} = \frac{p_{hn}}{q_n} \qquad (h = 0, 1, \ldots, N-1 ; n = 0, 1, 2, \ldots). \quad (12)$$

Then by (10) there is a positive constant c_3, such that for sufficiently large n

$$max\,(q_n, |p_{0n}|, |p_{1n}|, \ldots, |p_{N-1n}|) \leqq exp\,(c_3\, n\, (log\, n)^2). \quad . \quad (13)$$

Now by (3)

$$\alpha_n = \frac{1}{q_n} \sum_{h=0}^{N-1} p_{hn}\, \theta^h.$$

Hence Theorem 1 follows immediately from (13) and from the well known [6])

Theorem 4: To every real or complex algebraic number θ of degree N, there is a positive constant c_4, such that, if $A_0, A_1, \ldots, A_{N-1}$ are N integers which do not vanish simultaneously, then

$$\left| \sum_{h=0}^{N-1} A_h\, \theta^h \right| \geqq c_4\, \{max\,(|A_0|, |A_1|, \ldots, |A_{N-1}|)\}^{-(N-1)}.$$

Manchester, 8 November 1938.

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[6]) See J. F. KOKSMA, Diophantische Approximationen, Satz 6, p. 55.

Mathematics. — *Sur la répartition uniforme pour le module 1.* Par JEAN TEGHEM. (Communicated by Prof. J. G. VAN DER CORPUT.)

(Communicated at the meeting of December 17, 1938.)

Soient α et β deux nombres réels tels qu'on ait

$$\alpha < \beta \leqq \alpha + 1.$$

Soit S une suite infinie de domaines

$$D \ldots a_\mu \leqq x_\mu < b_\mu \qquad (\mu = 1, \ldots, m),$$

les lettres a_μ et b_μ désignant des nombres entiers. Soit $f(x) = f(x_1, \ldots, x_m)$ une fonction définie en tous les points x des domaines D, dont les coordonnées $x_1, \ldots, x_m$ sont entières.

On dit qu'un tel point x est solution entière de l'inégalité diophantienne

$$\alpha \leqq f(x) < \beta \qquad (mod.\ 1)$$

s'il existe une valeur entière y telle qu'on ait

$$\alpha \leqq f(x) - y < \beta.$$

Désignons par $A(D)$ le nombre des points x à coordonnées entières, situés dans un domaine D, et par $A_1(D)$ le nombre de ces points, qui sont solutions de l'inégalité diophantienne

$$0 \leqq f(x) < \gamma \qquad (mod.\ 1), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

γ étant donc un nombre positif, inférieur ou égal à 1. Nous supposons que $A(D)$ tend vers l'infini quand D parcourt la suite S. On dit que la fonction $f(x)$ est répartie uniformément pour le module 1 dans la suite S des domaines D lorsque, quel que soit le choix du nombre γ,

$$\frac{A_1(D)}{A(D)} \to \gamma$$

quand D parcourt la suite S.

Citons deux applications aisées d'un théorème de M. VAN DER CORPUT sur la répartition uniforme pour le module 1 [1]).

Application 1. Soit S une suite d'intervalles $a \leqq x < b$, telle qu'on ait

$$a \to \infty \quad , \quad \frac{b-a}{a} \to 1$$

lorsque l'intervalle parcourt la suite. Soit λ un nombre positif, *non entier*.

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[1]) J. G. VAN DER CORPUT. Diophantische Ungleichungen. I. Zur Gleichverteilung modulo Eins, Acta mathematica, **56**, page 440.

On peut affirmer que la fonction $x^\lambda \log x$ est répartie uniformément pour le module 1 dans les intervalles de la suite S.

On ne peut pas démontrer, par le théorème de M. VAN DER CORPUT, que la propriété est encore vraie lorsque λ est positif et *entier*.

Application 2. Soient $C_1, \ldots, C_m$ des nombres réels quelconques et $\lambda_1, \ldots, \lambda_m$ des nombres réels tels qu'on ait

$$\lambda_1 > 1 \ , \ \lambda_1 \not\equiv 0 \ (mod.\ 1) \ , \ \lambda_\mu < \lambda_1 \qquad (\mu = 2, \ldots . m);$$

soit Λ le plus petit nombre entier, qui soit supérieur à λ_1, et soient α et β deux nombres réels tels qu'on ait

$$\alpha < \beta \leqq \alpha + 1.$$

Considérons la fonction $q(x)$, et les fonctions $y_1(x)$ et $y_2(x)$, telles qu'on ait

$$q(x) = C_1 x^{\lambda_1} + \ldots + C_m x^{\lambda_m},$$

$$\lim_{x \to \infty} \{y_1(x) - q(x)\} = \alpha \quad , \quad \lim_{x \to \infty} \{y_2(x) - q(x)\} = \beta,$$

et désignons par $A_2(a, b)$ le nombre des solutions entières x, situées dans l'intervalle $a \leqq x < b$, de l'inégalité

$$y_1(x) < y < y_2(x), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

où y peut prendre n'importe quelle valeur entière.

M. VAN DER CORPUT démontre sur un exemple numérique que

$$\frac{A_2(a, b)}{b - a} \to \beta - \alpha$$

si a et b sont des nombres naturels tels qu'on ait

$$\frac{b - a}{a^{\Lambda - \lambda_1}} \to \infty \ ^{2}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Nous allons nous inspirer de la démonstration du théorème cité pour établir un théorème nouveau qui va nous permettre de renforcer les applications 1 et 2 par les deux suivantes.

Application 3. La propriété énoncée dans l'application 1 est encore vraie lorsque λ est positif et *entier*.

Application 4. La propriété énoncée dans l'application 3 est encore vraie lorsqu'on a

$$\frac{b - a}{a^{\frac{\Lambda - \lambda_1}{2}}} \to \infty \quad , \quad b = O(a), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

cette dernière notation signifiant que la quantité $\frac{b}{a}$ est bornée.

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2) Mémoire cité, p. 443.

La seconde condition (4) n'est pas fort gênante en ce qui concerne les cas nouveaux que la première condition (4) ajoute à (3). Elle est en effet remplie dès qu'on a

$$b-a = O(a^{\lambda_1 - \lambda_2}).$$

Nous aurons besoin, pour établir ce théorème, d'un théorème de M. VAN DER CORPUT [3]) sur les sommes DE WEYL généralisées, que nous citons comme lemme.

Lemme 1. *Si P est entier, $X \geqq 3$, $r > 0$ et $\Delta^2 f(x) \geqq r$ $(x = P+1, \ldots, P+X-2)$, alors on a* [4])

$$\left| \sum_{x=P+1}^{P+X} e^{2\pi i f(x)} \right| < 3r^{-\frac{1}{2}}(2 + \Delta f(P+X-1) - \Delta f(P+1)).$$

Il est à noter que $f(x)$ est ici une fonction d'une seule variable.

Prenons encore, comme lemmes, deux propriétés classiques du concept de répartition uniforme pour le module 1.

Lemme 2. *Une condition nécessaire et suffisante pour qu'une fonction $f(x) = f(x_1, \ldots, x_m)$ soit répartie uniformément pour le module 1 dans les domaines D d'une suite S est que, pour tout entier h non nul,*

$$\frac{1}{A(D)} \sum_{x \text{ dans } D} e^{2\pi i h f(x)} \to 0$$

quand D parcourt la suite S, la notation $\sum_{x \text{ dans } D}$ représentant la somme étendue à tous les points x à coordonnées entières du domaine D.

Lemme 3. *Supposons que la longueur $b_\sigma - a_\sigma$ d'au moins un des côtés du domaine D tende vers l'infini lorsque le domaine parcourt la suite S. Une condition suffisante pour qu'une fonction $f(x)$ soit répartie uniformément pour le module 1 dans les domaines D de la suite S est alors que, pour tout nombre naturel h, la fonction*

$$f_h(x) = f(x_1, \ldots, x_{\sigma-1}, x_\sigma + h, x_{\sigma+1}, \ldots, x_m) - f(x_1, \ldots, x_{\sigma-1}, x_\sigma, x_{\sigma+1}, \ldots, x_m)$$

soit répartie uniformément pour le module 1 dans les domaines

$$D_h \ldots a_\sigma \leqq x_\sigma < b_\sigma - h \ , \ a_\mu \leqq x_\mu < b_\mu \qquad (\mu = 1, \ldots, \sigma-1, \sigma+1, \ldots, m).$$

Théorème. *Soient $k_1, \ldots, k_m$ des entiers $\geqq 0$, avec $k_1 > 1$ et S une suite de domaines*

$$D \ldots a_\mu \leqq x_\mu < b_\mu \qquad (\mu = 1, \ldots, m),$$

a_μ et b_μ désignant des entiers tels que chaque côté $b_\mu - a_\mu$ croisse au delà

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[3]) Voir Proceedings, **40**, page 838 (1937).

[4]) Par définition, $\Delta f(x) = f(x+1) - f(x)$; $\Delta^k f(x) = \Delta(\Delta^{k-1} f(x))$.

de toute limite quand D parcourt la suite S. Si à chaque domaine D de la suite S correspondent deux nombres positifs r et R $(r \leqq R)$ tels qu'on ait

$$(b_1 - a_1)\sqrt{r} \to \infty \quad \text{et} \quad \frac{R}{\sqrt{r}} \to 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

quand D parcourt S, et si la fonction $f(x) = f(x_1, \ldots, x_m)$ (qui peut dépendre de D) est définie en tout point x à coordonnées entières de D de telle façon que la fonction

$$\Delta_1^{k_1} \ldots \Delta_m^{k_m} f(x) \quad [5])$$

ait, dans le domaine

$$D_k \ldots a_\mu \leqq x_\mu < b_\mu - k_\mu ,$$

une valeur ω ou bien constamment située dans l'intervalle $r \leqq \omega \leqq R$, on bien constamment située dans l'intervalle $-R \leqq \omega \leqq -r$, alors la fonction $f(x)$ est répartie uniformément pour le module 1 dans les domaines D.

Remarque. Le théorème cité de M. VAN DER CORPUT a un énoncé ressemblant très fort à celui-ci. La première condition (5) y est remplacée par une condition plus restrictive; la seconde condition (5) est, par contre, remplacée par une condition moins restrictive. Le théorème de M. VAN DER CORPUT suppose en outre que $\Delta_1^{k_1} \ldots \Delta_m^{k_m} f(x)$ est monotone dans les domaines D_k.

Démonstration. Deux cas sont à considérer.

1^0. $k_1 + \ldots + k_m = 2$. Donc $k_1 = 2$ et $k_\mu = 0$ $(\mu = 2, \ldots, m)$.

$f(x)$ étant considérée comme fonction de x_1, le lemme 1 et les conditions (5) donnent

$$\left.\begin{aligned} \left| \frac{1}{A(D)} \sum_{x \text{ dans } D} e^{2\pi i f(x)} \right| &\leqq \frac{1}{A(D)} \sum_{x_2=a_2}^{b_2-1} \ldots \sum_{x_m=a_m}^{b_m-1} \left| \sum_{x_1=a_1}^{b_1-1} e^{2\pi i f(x)} \right| \\ \leqq \frac{1}{A(D)} \sum_{x_2=a_2}^{b_2-1} \ldots \sum_{x_m=a_m}^{b_m-1} &\left\{ \frac{3}{\sqrt{r}} \left(2 + \Delta_1 f(b_1-2) - \Delta_1 f(a_1)\right) \right\} \\ < \frac{3 \cdot 2}{(b_1-a_1)\sqrt{r}} + \frac{3(b_1-a_1)R}{(b_1-a_1)\sqrt{r}} &= \frac{6}{(b_1-a_1)\sqrt{r}} + 3\frac{R}{\sqrt{r}} \to 0 \end{aligned}\right\} \quad . \quad . \quad (6)$$

Soit h un nombre entier quelconque, non nul. Les hypothèses du théorème subsistent quand la fonction $f(x)$ est remplacée par $hf(x)$, r par $|h|r$ et R par $|h|R$. Il résulte donc de (6) que l'on a

$$\frac{1}{A(D)} \sum_{x \text{ dans } D} e^{2\pi i h f(x)} \to 0$$

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[5]) $\Delta_1^{k_1} \ldots \Delta_m^{k_m} f(x) =$

$$\sum_{\chi_1=0}^{k_1} \ldots \sum_{\chi_m=0}^{k_m} (-1)^{k_1+\ldots+k_m-\chi_1-\ldots-\chi_m} \binom{k_1}{\chi_1} \ldots \binom{k_m}{\chi_m} f(x_1+\chi_1, \ldots, x_m+\chi_m).$$

quand D parcourt la suite S. La fonction $f(x)$ est donc répartie uniformément pour le module 1 dans les domaines D, en vertu du lemme 2.

2^0. $k_1 + \ldots + k_m > 2$. Nous pouvons supposer le théorème démontré lorsque $k_1 + \ldots + k_m$ est remplacé par un nombre entier $\geqq 2$, plus petit. Définissons le nombre σ de la façon suivante: si k_1 est $\geqq 3$, soit $\sigma = 1$; si $k_1 = 2$, soit $\sigma \geqq 2$, choisi de telle sorte que k_σ soit positif. Soit h un nombre naturel quelconque; posons

$$k'_\sigma = k_\sigma - 1 \; ; \; a'_\sigma = a_\sigma \; ; \; b'_\sigma = b_\sigma - h \; ;$$
$$k'_\mu = k_\mu \quad ; \; a'_\mu = a_\mu \; ; \; b'_\mu = b_\mu \qquad (\mu = 1, \ldots, m \; ; \; \mu \neq \sigma).$$

Soit D' le domaine

$$a'_\mu \leqq x_\mu < b'_\mu.$$

On constate que les hypothèses du théorème restent vérifiées quand k_μ est remplacé par k'_μ, a_μ par a'_μ, b_μ par b'_μ, D par D', r par $|h|r$, R par $|h|R$, et $f(x)$ par

$$f_h(x) = f(x_1, \ldots, x_{\sigma-1}, x_\sigma + h, x_{\sigma+1}, \ldots, x_m) - f(x_1, \ldots, x_{\sigma-1}, x_\sigma, x_{\sigma+1}, \ldots, x_m).$$

La fonction $f_h(x)$ est donc, en vertu du théorème à démontrer appliqué avec $k_\sigma - 1$ au lieu de k_σ, répartie uniformément pour le module 1 dans les domaines D'. Il en est donc de même, dans les domaines D, de la fonction $f(x)$, en vertu du lemme 3.

Tirons maintenant de ce théorème les conséquences annoncées.

Application 3. *Démonstration.* Supposons donc que λ soit un nombre naturel. Nous voyons aisément que l'on a pour la fonction $f(x) = x^\lambda \log x$

$$\lim_{x \to \infty} \frac{x \triangle^{\lambda+1} f(x)}{\lambda !} = 1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Il en résulte, si nous désignons respectivement par R et r le maximum et le minimum de $\triangle^{\lambda+1} f(x)$ dans l'intervalle $a \leqq x < b$, que

$$\sqrt{R} \to 0 \quad , \quad (b-a)\sqrt{r} \to \infty$$

lorsque l'intervalle parcourt la suite considérée. Mais la quantité $\frac{R}{r}$ est bornée, en vertu de (7) et de ce que $b = O(a)$. Nous en déduisons que

$$\frac{R}{\sqrt{r}} \to 0 \quad , \quad (b-a)\sqrt{r} \to \infty \, ,$$

et nous pouvons donc affirmer que la fonction $f(x) = x^\lambda \log x$ est répartie uniformément pour le module 1 dans la suite d'intervalles $a \leqq x < b$ considérée.

Application 4. *Démonstration.* Il résulte de (4) que

$$\frac{b^{\frac{\Lambda-\lambda_1}{2}}}{b-a} \to 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Considérons les deux inégalités auxiliaires

$$-\beta+\delta < q(x) < -\alpha-\delta \qquad (mod.\ 1), \quad . \quad . \quad . \quad . \quad (9)$$

$$-\beta-\delta < q(x) < -\alpha+\delta \qquad (mod.\ 1), \quad . \quad . \quad . \quad . \quad (10)$$

δ étant un nombre positif, inférieur à $\frac{\beta-\alpha}{2}$. Il existe un nombre positif c, dépendant seulement de δ, tel qu'on ait, pour tout couple de nombres entiers g et b $(c \leq g < b)$,

$$A_9(g,b) \leqq A_2(g,b) \leqq A_{10}(g,b), \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$A_9(g,b)$, $A_2(g,b)$ et $A_{10}(g,b)$ désignant respectivement les nombres de solutions entières x, situées dans l'intervalle $g \leqq x < b$, des inégalités (9), (2), (10).

Les intervalles $a \leqq x < b$, pour lesquels on a $b \leqq c$, ne sont qu'en nombre fini ,et ne doivent pas être considérés. Associons à chacun des autres intervalles $a \leqq x < b$ un nombre entier g, supérieur ou égal à a et à c, inférieur à b et tel qu'on ait

$$\frac{g-a}{b-a} \to 0 \qquad \left(\text{donc } \frac{b-g}{b-a} \to 1\right) . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Il découle de (11) qu'on a

$$A_9(g,b)-(g-a) \leqq A_2(a,b) \leqq A_{10}(g,b)+(g-a). \quad . \quad . \quad (13)$$

On a d'autre part, en vertu de (12) et (8),

$$\frac{b-g}{b^{\frac{\Lambda-\lambda_1}{2}}} = \frac{b-g}{b-a} \cdot \frac{b-a}{b^{\frac{\Lambda-\lambda_1}{2}}} \to \infty . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Quant à la fonction $\Delta^\Lambda q(x)$, elle est telle que

$$\lim_{x\to\infty} \frac{\Delta^\Lambda q(x) \cdot x^{\Lambda-\lambda_1}}{C_1 \lambda_1 (\lambda_1-1) \ldots (\lambda_1-\Lambda+1)} = 1. \quad . \quad . \quad . \quad . \quad (15)$$

Désignons respectivement par r et R le minimum et le maximum de $|\Delta^\Lambda q(x)|$ dans l'intervalle $g \leqq x < b-\Lambda$. Les formules (14) et (15) montrent que

$$(b-g)\sqrt{r} = \frac{b-g}{b^{\frac{\Lambda-\lambda_1}{2}}} \cdot b^{\frac{\Lambda-\lambda_1}{2}} \sqrt{r} \to \infty$$

quand l'intervalle $a \leqq x < b$ parcourt la suite considérée. Nous avons également, en vertu de la seconde condition (4),

$$\frac{R}{\sqrt{r}} \to 0.$$

La fonction $q(x)$ est donc répartie uniformément pour le module 1 dans les intervalles $g \leqq x < b$, de telle sorte qu'on a

$$\frac{A_9(g, b)}{b-g} \to (\beta-\alpha)-2\delta \quad , \quad \frac{A_{10}(g, b)}{b-g} \to (\beta-\alpha)+2\delta. \quad . \quad . \quad (16)$$

De (13), (12) et (16) suit alors qu'on a

$$\textit{lim. sup.} \frac{A_2(a, b)}{b-a} \leqq (\beta-\alpha)+2\delta \quad , \quad \textit{lim. inf.} \frac{A_2(a, b)}{b-a} \geqq (\beta-\alpha)-2\delta.$$

Il est donc ainsi démontré qu'on a

$$\frac{A_2(a, b)}{b-a} \to \beta-\alpha.$$

Chemistry. — *Strömungspotentiale und Oberflächenleitfähigkeit.* II. Von A. J. RUTGERS und ED. VERLENDE. (Mitteilung aus dem physikalisch-chemischen Laboratorium der Reichsuniversität Gent, Belgien). (Communicated by Prof. H. R. KRUYT).

(Communicated at the meeting of December 17, 1938.)

ZUSAMMENFASSUNG.

Es wird der elektrostatische Potentialverlauf in einer Flüssigkeit an einer ebenen hochgeladenen Wand ($\varepsilon \psi >> k\,T$) theoretisch behandelt und dabei die Annahme gemacht, dass im gamzen Konzentrationsgebiet l, die Ladung der diffusen Schicht, konstant bleibt; auf diese Weise ist es möglich, ζ als Funktion der Konzentration zu berechnen; es ergibt sich, dass die ζ — *log c*-Kurve eine Gerade ist, deren Neigung 58 mV. beträgt; weiter, dass die "Dicke" der diffusen Schicht nur schwach von der Konzentration abhängig ist.

Die Vergleichung mit der Erfahrung fällt befriedigend aus; die experimentelle ζ — *log c*-Kurve ist nahezu eine Gerade; ihre Neigung beträgt 54 mV. Für die Ladung/cm^2 der diffusen Schicht l wird 1740 e.s.e./cm^2 gefunden; die Kapazität der diffusen Doppelschicht bei $c = 0{,}8 \,.\, 10^{-4}$ Mol/L beträgt 4,25 μF/cm^2; dem entspricht eine "Dicke" $d = 1{,}7 \,.\, 10^{-6}$ cm.

Für die Erklärung der experimentellen Oberflächenleitfähigkeit reicht die Ladungsdichte von 1740 e.s.e./cm^2 nicht aus; dazu braucht man mindestens 50.000—150.000 e.s.e./cm^2.

§ 1. Mit Hilfe der HELMHOLTZ—SMOLUCHOWSKIschen Gleichung

$$\zeta = \frac{4\pi\eta}{D}\,\sigma\,\frac{E}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

ist est möglich, aus Messungen des Strömungspotentiales E das elektrokinetische Potential ζ zu bestimmen; die so von verschiedenen Forschern [1]) erhaltenen ζ-Werte zeigten in ihrer Konzentrationsabhängigkeit oft einen Anstieg bei kleinen Konzentrationen; diesen Anstieg haben wir in einer vorigen Abhandlung [2]) als eine Folge der Nichtberücksichtigung des Umstandes gedeutet, dass σ in der Gl. (I) nicht die Leitfähigkeit der untersuchten Flüssigkeit, sondern die Leitfähigkeit *dieser Flüssigkeit in der Kapillare* bedeutet, dass also die Oberflächenleitfähigkeit mitberücksichtigt werden muss. Zu unserem Bedauern haben wir dabei übersehen, dass BIKERMAN [3]) schon vor uns in einer theoretischen Arbeit diesen Anstieg der ζ—*c*-Kurve in genau derselben Weise gedeutet hatte.

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[1]) H. R. KRUYT, Proc. Kon. Akad. v. Wetensch., Amsterdam, **17**, 615 (1914); **19**, 1021 (1917).

H. FREUNDLICH und G. ETTISCH, Z. phys. Ch. **116**, 401 (1925).

[2]) A. J. RUTGERS, ED. VERLENDE und MA. MOORKENS, Proc. Kon. Ned. Akad. v, Wetensch., Amsterdam, **41**, 763 (1938).

[3]) J. J. BIKERMAN, Koll. Zeitschr., **72**, 100 (1935).

Durch Messungen an drei Kapillaren aus Jenaglas 16^{III} von verschiedenen Radien haben wir versucht, die Richtigkeit dieses Erklärungsversuches zu beweisen; das Resultat war in Uebereinstimmung mit der Erwartung; weiter zeigte sich, dass der „wahre" ζ-Wert für reines Wasser sich als mehr als 225 mV. ergab, während Messungen an engen Kapillaren auf einen scheinbaren ζ-Wert von 75 mV. führten.

Wir wollen jetzt versuchen, die für KCl-Lösungen erhaltene ζ—c-Kurve theoretisch zu erfassen.

§ 2. Die Zusammenfassung der POISSONschen und BOLTZMANNschen Gleichungen

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi n_0 \varepsilon}{D}\left[e^{-\frac{\varepsilon\psi}{kT}} - e^{\frac{\varepsilon\psi}{kT}}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

ergibt bei einmaliger Integration:

$$\left(\frac{d\psi}{dx}\right)^2 + C_1 = \frac{8\pi n_0 kT}{D}\left[e^{\frac{\varepsilon\psi}{kT}} + e^{-\frac{\varepsilon\psi}{kT}}\right].$$

Für $x = \infty$, gilt: $\psi = 0$; $\frac{d\psi}{dx} = 0$; also:

$$C_1 = \frac{8\pi n_0 kT}{D} 2$$

$$\left(\frac{d\psi}{dx}\right)^2 = \frac{8\pi n_0 kT}{D}\left[e^{\frac{\varepsilon\psi}{kT}} + e^{-\frac{\varepsilon\psi}{kT}} - 2\right] \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Wurzelziehen und Wahl des negativen Vorzeichens der Wurzel ergibt:

$$\frac{d\psi}{dx} = -\sqrt{\frac{8\pi n_0 kT}{D}}\left[e^{\frac{\varepsilon\psi}{2kT}} - e^{-\frac{\varepsilon\psi}{2kT}}\right] = -\sqrt{\frac{32\pi n_0 kT}{D}}\, sh\, \frac{\varepsilon\psi}{2kT}. \quad (4)$$

Nun ist:

$$\int \frac{d\alpha}{sh\,\alpha} = ln\, th\, \frac{\alpha}{2} + C' = ln\, \frac{e^{\alpha} - 1}{e^{\alpha} + 1} + C'.$$

Also:

$$ln\, \frac{e^{\frac{\varepsilon\psi}{2kT}} - 1}{e^{\frac{\varepsilon\psi}{2kT}} + 1} + C' = -\sqrt{\frac{8\pi n_0 \varepsilon^2}{DkT}} \cdot x = -\varkappa x \quad . \quad . \quad . \quad . \quad (5)$$

wo

$$\varkappa = \sqrt{\frac{8\pi n_0 \varepsilon^2}{DkT}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$\frac{1}{\varkappa}$ ist im Falle, dass $\frac{\varepsilon\psi}{kT} \ll 1$, die bekannte „Dicke" der diffusen Schicht; für den uns hier interessierenden Fall, wo $\frac{\varepsilon\psi}{kT} \gg 1$, hat $\frac{1}{\varkappa}$ keine anschauliche Bedeutung; wohl aber können wir auch hier eine „Dicke" der diffusen Schicht berechnen.

Aus der Gl. (5) folgt:

$$\frac{e^{\frac{\varepsilon\psi}{2kT}} - 1}{e^{\frac{\varepsilon\psi}{2kT}} + 1} = Ce^{-\varkappa x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Oder:

$$\frac{\varepsilon\psi}{2kT} = ln\,\frac{1 + Ce^{-\varkappa x}}{1 - Ce^{-\varkappa x}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Die Konstante C ist für eine gegebene Konzentration konstant, für verschiedene Konzentrationen verschieden.

Für $x = 0$ sei $\psi = \psi_0$:

$$\frac{\varepsilon\psi_0}{2kT} = ln\,\frac{1 + C}{1 - C} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

Also:

$$C = \frac{e^{\frac{\varepsilon\psi_0}{2kT}} - 1}{e^{\frac{\varepsilon\psi_0}{2kT}} + 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Durch die Gleichungen (8), (9) und (6) ist ψ vollständig bestimmt. Unser Endresultat ist in völliger Uebereinstimmung mit der Gl. (9) von GOUY [4]), der dasselbe Problem in etwas verschiedener Weise behandelt hat.

Wir wollen nun durch eine physikalische Annahme die „Konstante" C als Funktion der Konzentration festlegen. Die Ebene $x = 0$ sei die Ebene, wo die Flüssigkeit eben anfängt, zu bewegen. Jetzt gilt:

$$\zeta = \psi\,(x = 0) = \psi_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

wo wir mit ζ natürlich das „wahre" ζ meinen, das in der vorigen Arbeit mit ζ_0 angegeben war.

Wir nehmen nun an, dass im ganzen untersuchten Konzentrationsgebiet 1, die totale diffuse Ladung/cm^2 in der Flüssigkeit, konstant bleibt; es liegt ja vor der Hand, sich von der Wirkung des hinzugefügten Elektrolyten folgendes Bild zu machen:

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[4]) M. GOUY, Journal de Physique, 4^0 Série, **9**, 457 (1910).

1. Durch eine geringe Quantität indifferentes Elektrolyt wird die diffuse Schicht nur eingedrückt.

2. Erst nachdem grössere Quantitäten indifferentes Elektrolyt hinzugefügt worden sind, der „Schwerpunkt" der diffusen Ladung sich der Wand in beträchtlicher Weise genähert hat, fängt bei noch weiter gehendem Eindrücken der diffusen Schicht die diffuse Ladung an, die Ebene $x = 0$ zu durchschreiten.

Unsere Annahme besagt also, dass für die von uns untersuchten Konzentrationen von $0-80 \cdot 10^{-6}$ Aeq./L die zweite Erscheinung noch nicht in Betracht gezogen zu werden braucht.

Die Uebereinstimmung zwischen Theorie und Erfahrung, die sich ergeben wird, ergibt eine weitere Rechtfertigung dieser Annahme.

(Auch in der Arbeit von GOUY (l.c. S. 467) wird diese Annahme schon gemacht; auch in einer Arbeit von VERWEY (Diss. Utrecht 1934, S. 80); für ein andres Problem, in einer Arbeit von MÜLLER (Koll. Beih. 26, 257, 1928)).

Wir können nun die Grösse C ausdrücken in der konstanten Grösse 1, und der veränderlichen Grösse $\varkappa$.

Für 1 gilt:

$$l = \int_0^\infty \varrho \, dx = -\int_0^\infty \frac{D}{4\pi} \frac{d^2\psi}{dx^2} \, dx = \frac{D}{4\pi} \left(\frac{d\psi}{dx}\right)_0 \quad . \quad . \quad . \quad . \quad (11)$$

Aus der Gl. (8) folgt:

$$\frac{\varepsilon}{2kT} \frac{d\psi}{dx} = -\frac{2\varkappa C e^{-\varkappa x}}{1 - C^2 e^{2\varkappa x}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$l = \frac{D}{4\pi} \left(\frac{d\psi}{dx}\right)_0 = -\frac{DkT\varkappa}{\pi\varepsilon} \frac{C}{1-C^2} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Um zu grosse Komplikation in den Formeln zu vermeiden, gehen wir auf eine Näherungsrechnung über:

Aus der Gl. (8a) folgt:

$$C = \frac{e^{\frac{\varepsilon\psi_0}{2kT}} - 1}{e^{\frac{\varepsilon\psi_0}{2kT}} + 1} = -\frac{1 - e^{\frac{\varepsilon\psi_0}{2kT}}}{1 + e^{\frac{\varepsilon\psi_0}{2kT}}} = -(1-\delta) \quad . \quad . \quad . \quad . \quad (14)$$

wo beachtet ist, dass in unserm Fall (Glass-Wasser) ψ kräftig negativ ist, und $\left|\frac{\varepsilon\psi}{2kT}\right| \gg 1$ ist $\left(\text{also } |\psi| \gg \frac{2kT}{\varepsilon} \text{ oder } |\psi| \gg 50 \text{ mV}\right)$.

Die Grösse δ ist eine kleine positive Grösse; für l gilt jetzt auf Grund von Gl. (12) und (13):

$$l = \frac{DkT\varkappa}{\pi\varepsilon} \frac{1-\delta}{1-(1-\delta)^2} \sim \frac{DkT\varkappa}{2\pi\varepsilon\delta} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Also:

$$\delta = \frac{D k T \varkappa}{2 \pi \varepsilon l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Und:

$$C = -(1 - \delta) = -\left(1 - \frac{D k T \varkappa}{2 \pi \varepsilon l}\right) \quad . \quad . \quad . \quad . \quad . \quad (17)$$

In demjenigen Gebiet, wo $|\psi| \gg 25$ mV. ist, wird also das Potential $\psi(x)$ durch die Gl. (8), (17), und den Wert von l (s. u.) bestimmt.

Aus den Gl. (8) und (17) folgt für $x = 0$:

$$\psi_0 = \zeta = \frac{2 k T}{\varepsilon} ln \frac{1 + C}{1 - C} \simeq \frac{2 k T}{\varepsilon} ln \frac{\delta}{2} = \frac{2 k T}{\varepsilon} ln \frac{D k T \varkappa}{4 \pi \varepsilon l} \quad . \quad . \quad (18)$$

Die Konzentrationsabhängigkeit von ζ kommt durch das Auftreten von $\varkappa$ zustande.

Die Dicke d der Doppelschicht identifizieren wir mit dem x-Wert der Schwerpunktebene der diffusen Ladung. Hierfür gilt nach GOUY:

$$d = \frac{1}{l} \int_0^\infty x \varrho(x) \, dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Für ϱ, die Ladungsdichte, gilt nach der POISSONschen Gleichung:

$$\varrho(x) = -\frac{D}{4\pi} \frac{d^2 \psi}{dx^2} .$$

Dann gilt:

$$d = -\frac{D}{4\pi l} \int_0^\infty x \frac{d^2 \psi}{dx^2} \, dx = -\frac{D}{4\pi l} \left\{ \left[x \frac{d\psi}{dx} \right]_0^\infty - \int_0^\infty \frac{d\psi}{dx} \, dx \right\} =$$

$$= \frac{D}{4\pi l} \left[\psi \right]_0^\infty = -\frac{D \psi_0}{4\pi l} \quad . \quad . \quad . \quad . \quad (20)$$

Oder auch:

$$d = -\frac{\psi_0}{\left(\frac{d\psi}{dx}\right)_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Die Dicke d der diffusen Schicht ändert sich mit der Konzentration nach Gl. (20) (bei konstant angenommenem l) wie ψ_0, also sehr langsam.

Wir wollen jetzt die Gl. (18) ausarbeiten. Wir schreiben:

$$\varkappa = \sqrt{\frac{8 \pi \varepsilon^2 n_0}{D k T}} = \sqrt{\frac{8 \pi \varepsilon^2 N c}{1000 D k T}} = \alpha \sqrt{c} \quad . \quad . \quad . \quad . \quad (22)$$

(N AVOGADROsche Zahl, c Konzentration in Mol/L), wo

$$a = 3{,}3 \cdot 10^7 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Also, nach Gl. (18):

$$\left.\begin{aligned} \zeta &= \frac{2kT}{0{,}4343\,\varepsilon}\left[\log\frac{DkT}{4\pi\varepsilon} + \log a - \log l + \tfrac{1}{2}\log c\right] \\ \zeta &= 116\,[-3{,}28 + 7{,}52 - \log l + \tfrac{1}{2}\log c] = 116\,[4{,}24 - \log l] + 58\log c. \end{aligned}\right\} \quad (24)$$

Hier ist ζ schon ausgedrückt in mV.

Unsere Betrachtung ergibt also:

1. Die ζ — $\log c$-Kurve ist eine Gerade.
2. Die Neigung dieser Geraden beträgt 58 mV.
3. Es ist möglich, die diffuse Ladung/cm², l, zu berechnen.

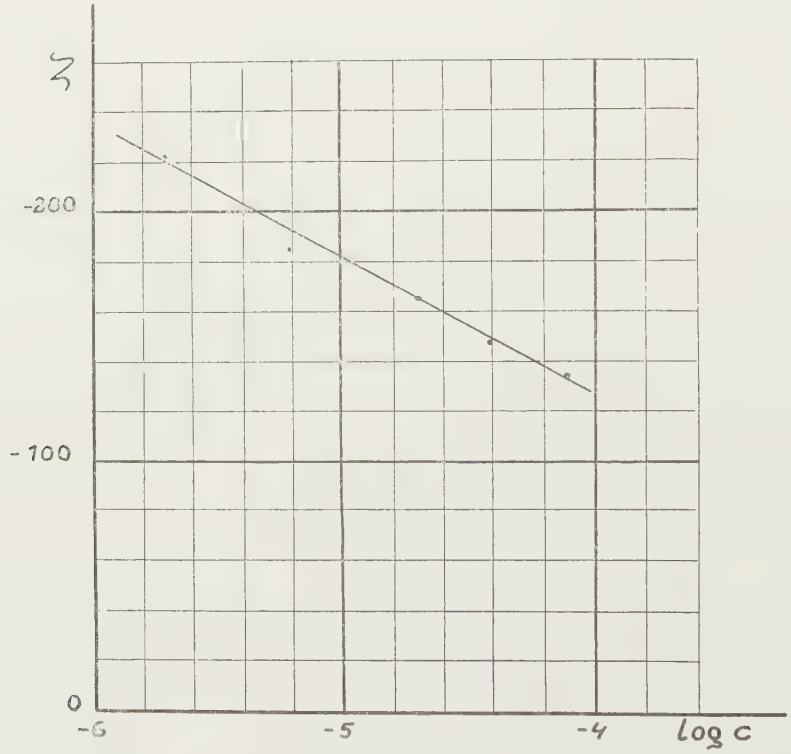

Fig. 1. ζ als Funktion von $\log c$.

§ 3. In der nachstehenden Tabelle findet man die in der vorigen Arbeit [5]) erhaltenen wahren ζ-Werte in mV.; in der Figur ist ζ gegen $\log c$ aufgetragen; man sieht:

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[5]) A. J. RUTGERS, ED. VERLENDE und MA. MOORKENS, l.c. S. 772.

1. Die ζ —*log c*-Kurve weicht nicht sehr von einer Geraden ab.
2. Die Neigung dieser Geraden beträgt 54 mV., in befriedigender Uebereinstimmung mit dem theoretischen Wert.
3. Der ζ-Wert für $\log c = -6$ beträgt -232 mV.

TABELLE 1.

c	$\log c$	ζ in mV	σ_w in mho
$2 \cdot 10^{-6}$	0,30—6	—222	$1{,}13 \cdot 10^{-8}$
$6 \cdot 10^{-6}$	0,78—6	—184	$1{,}04 \cdot 10^{-8}$
$20 \cdot 10^{-6}$	1,30—6	—165	$1{,}57 \cdot 10^{-8}$
$40 \cdot 10^{-6}$	1,60—6	—147	$1{,}79 \cdot 10^{-8}$
$80 \cdot 10^{-6}$	1,90—6	—136	$3{,}18 \cdot 10^{-8}$

Also wegen:

$$\frac{\zeta}{116} = 4{,}24 - \log l + \tfrac{1}{2} \log c$$

$$\log l = 4{,}24 - \frac{\zeta}{116} + \tfrac{1}{2} \log c = 4{,}24 + 2 - 3 = 3{,}24$$

$$l = 1740 \text{ e. s. e./cm}^2.$$

Die Ladung der diffusen Schicht beträgt also 1740 e.s.e./cm², d. h. $3{,}6 \cdot 10^{12}$ Elementarladungen/cm².

Für die Kapazität/cm² der diffusen Doppelschicht finden wir noch:

Bei $c = 10^{-6}$ *Mol/L*: $C = \frac{l}{V} = \frac{1740}{3.10^9 \cdot 0{,}232} = 2{,}45 \cdot 10^{-6} F = 2{,}45\ \mu F.$

Bei $c = 0{,}8 \cdot 10^{-4}$ *Mol/L*: $C = \frac{l}{V} = \frac{1740}{3.10^9 \cdot 0{,}136} = 4{,}25 \cdot 10^{-6} F = 4{,}25\ \mu F.$

Durch die immer auftretenden Spuren Verunreinigung in destilliertem Wasser entspricht die letzte Lösung eher der Konzentration von $0{,}8 \cdot 10^{-4}$ Mol/L als die erste der Konzentration von $1 \cdot 10^{-6}$ Mol/L. Mit der letzten Lösung wollen wir uns also noch befassen. Der Wert der Kapazität entspricht folgender Dicke der diffusen Schicht:

$$d = \frac{D}{4\pi C} = \frac{80}{4\pi\, 4{,}25 \cdot 10^{-6} \cdot 9.10^{11}} = 1{,}7 \cdot 10^{-6} \text{ cm}.$$

Die Gleichung (20) hätte ergeben:

$$d = -\frac{D\psi_0}{4\pi l} = \frac{80 \cdot 0{,}136}{4\pi \cdot 1740 \cdot 300} = 1{,}66 \cdot 10^{-6} \text{ cm}.$$

Der Ladung/cm² der diffusen Schicht entspricht bei Annahme freier

Ionenbeweglichkeit, also einer Ionengeschwindigkeit von 6.10^{-4} cm. $sec^{-1}/V.cm^{-1}$ einer Wandleitfähigkeit σ_w gegeben durch

$$\sigma_w = \frac{1740}{3.10^9} . 6 . 10^{-4} = 3{,}5 . 10^{-10}\ mho.$$

Dieser Wert ist 30—100 mal kleiner als der experimentelle Wert von dem in dem von uns untersuchten Konzentrationsgebiet; d. h., es gibt bei negativen Werten von x, also zwischen der Ebene $x=0$ und der Wand, frei bewegliche (eventuell positive *und* negative) Ladungen von 50.000—150.000 e.s.e./cm^2 (1—3.10^{14} Elementarladungen/cm^2).

Bemerkung während der Korrektur: Wenn wir uns beschränken auf das Problem, ψ_0 (oder ζ) als Funktion der Elektrolytkonzentration zu berechnen, so folgt aus den Gl. (3) und (11) sofort:

$$\left(\frac{4\pi l}{D}\right)^2 = \frac{8\pi n_0 k T}{D}\left[e^{\frac{\varepsilon\psi_0}{kT}} + e^{-\frac{\varepsilon\psi_0}{kT}} - 2\right] \simeq \frac{8\pi n_0 k T}{D} e^{-\frac{\varepsilon\psi_0}{kT}}$$

$\left(\text{wenn } \psi_0 \text{ negativ ist, und } \left|\frac{\varepsilon\psi_0}{kT}\right| \gg 1\right)$. Daraus folgt:

$$\psi_0 = \zeta = \frac{kT}{0{,}4343\,\varepsilon} log \frac{D k T n_0}{2\pi l^2}$$

in Uebereinstimmung mit den Gl. (18) und (6).

Jetzt können wir auch sofort die Lösung dieses Problems für einen $z_+ - z_-$-wertigen Elektrolyten hinschreiben; ein Molekül dieses Elektrolyten spalte sich in v_+ Kationen und v_- Anionen; dann gilt:

$$z_+ v_+ = z_- v_- = z v.$$

Anstatt der Gl. (2) kommt nun:

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi n_0 \varepsilon v z}{D}\left[e^{-\frac{z_+ \varepsilon\psi}{kT}} - e^{\frac{z_- \varepsilon\psi}{kT}}\right].$$

Anstatt der Gl. (3), und nach Benutzung der Gl. (11):

$$l^2 = \frac{D k T v z n_0}{2\pi}\left[\frac{1}{z_+} e^{-\frac{z_+ \varepsilon\psi_0}{kT}} + \frac{1}{z_-} e^{\frac{z_- \varepsilon\psi_0}{kT}} - \left(\frac{1}{z_+} + \frac{1}{z_-}\right)\right].$$

(Diese Gleichung wurde schon benutzt von VERWEY, Diss. Utrecht 1934, S. 80). Für unsere Versuchsbedingungen (negative, hochgeladene Wand) gilt angenähert:

$$\psi_0 = \zeta = \frac{kT}{0{,}4343\,\varepsilon}\frac{1}{z_+} log \frac{D k T v_+ n_0}{2\pi l^2} = \frac{58}{z_+} log \frac{D R T v_+ c}{2000\pi l^2}.$$

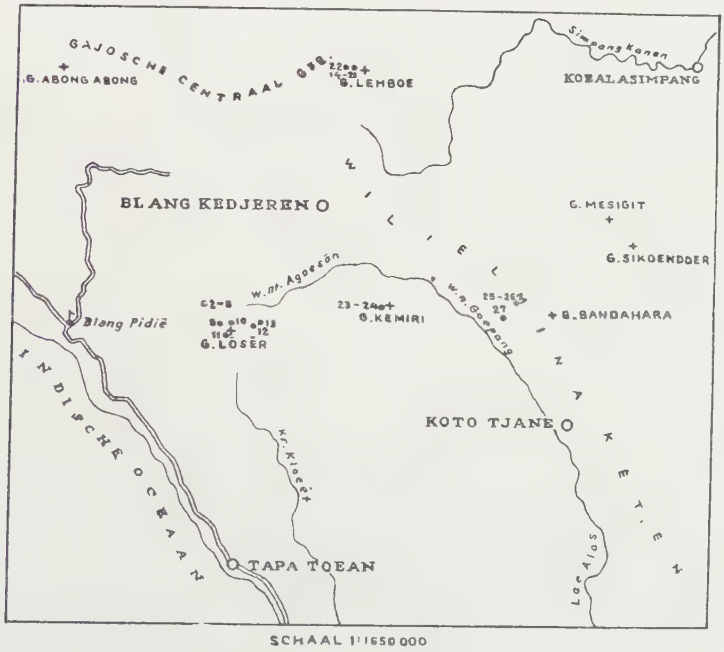
Hier ist c die Konzentration des Elektrolyten in *Mol/L*; ζ ist ausgedrückt in mV.; für 1—1, 2—1, 3—1, 4—1-wertige Elektrolyten sind die $\zeta - log\, c$ Kurven also Geraden, deren Neigung bzw. 58, 29, 19,3 und 14,5 mV. beträgt.

Geologie. — *Petrografisch en Mineralogisch Onderzoek van enkele gesteenten en zanden van den Gajo Loeeus (Atjeh).* Door F. A. VAN BAREN (Buitenzorg). (Communicated by Prof. L. RUTTEN.)

(Communicated at the meeting of December 17, 1938.)

Tijdens de expeditie, die in Jan.—Maart 1937 den Gajo Loeeus bezocht, werden door den leider Dr. VAN STEENIS (8) ook eenige gesteente- en zandmonsters verzameld. Het kwam gewenscht voor, dit materiaal te onderzoeken, omdat hierdoor een, zij het ook uiterst fragmentaire kennis van het nooit eerder bezochte gebied kon worden verkregen.

In het binnenland van Atjeh treft men hoofdzakelijk Praetertiair en Palaeogeen aan. Het Neogeen ontbreekt, terwijl de Kwartaire, vulkanische activiteit in het onderhavige gebied gering geweest is. De G. Losir, waarvan de vulkanische aard op grond van rolsteenen (VOLZ in 10) werd verondersteld, blijkt geen vulkaan te zijn. Slechts een enkel brok uitvloeiïngsgesteente werd aangetroffen; de hoofdmassa bestaat uit zandsteenen en kleischalies. Hetzelfde geldt voor den G. Banda Hara (9), dien VOLZ eveneens als een oude andesietvulkaan beschouwde. Het Praetertiair bestaat uit grauwacken, schalies, zandsteenen en kwartsieten. Discordant hierop begint het Palaeogeen met een basale breccie of conglomeraat, gevolgd door zandsteenen en dan door de zoogen. „zwarte kleisteen".



No. 2 (60157), 9 (60163). Zandsteenen, resp. Losir Bivak 3 en 5. Losse, vrij fijnkorrelige gesteenten. O.h.m.: troebele, veelal unduleus uitdoovende, kleine kwartskorrels, met muscoviet en toermalijn als accessorische mineralen. In no. 9 ook zirkoon en kleine fragmenten van zeer fijnkorrelige kwartsgesteenten.

No. 11 (60164). Plagioklaasbasalt, kamlijn boven Losir bivak 6. Grauw, compact porfierisch gesteente met fenokristen van veldspaat en pyroxeen. O.h.m.: grondmassa van veldspaten, erts, serpentijnachtige verweeringsproducten en iets vulkanisch glas; groote fenokristen van labradoriet (64—70 % An), monokliene augiet en geserpentiniseerde olivijn.

No. 12. Contact-hydrothermaalmetamorphe pyroxeen-amphiboolandesiet [1]). Steenhelling van grooten rug naar Lau Alas, circa 3300 m b. z. Grijs, porfierisch gesteente met veldspaat-fenokristen. O.h.m.: intermediaire plagioklaas, ten deele omgezet in aggregaten epidoot en albiet; pyroxeen geheel omgezet in aggregaten epidoot (en prehniet), amphibool slechts herkenbaar aan ruitvormige omtrekken en smallen opacietzoom; omgezet in serpentijn. Fijnkristallijne grondmassa van veldspaat, epidoot, serpentijn, ertsspikkels en ook wat secundaire resten van sulfidisch erts, prehniet en kwarts.

No. 13. Contactmetamorfe amphibool-epidoot-kwartsiet, kamp 8, groote rug naar de Lau Alas. Geelbruin gesteente. O.h.m.: gerekristalliseerde kwartskorrels, nematoblastische actinoliet, korrelige aggregaten van zoisiet en epidoot, met accessorische secundaire, idiomorfe toermalijn (violet-kleurloos), zirkoon en biotiet. De gesteenten (12) en (13) doen vermoeden, dat er in de buurt een zure intrusie aanwezig is, aan welke de omzettingen zijn toe te schrijven.

No. 14 (60165) en 20 (60171). Kleischalies, top Goh Lemboeh. Dichte, grijze gesteenten. O.h.m.: sericietvilt met kwartskorreltjes; in no. 14 sporadische rutielnaaldjes, in no. 20 chlorietachtige schubjes.

No. 15 (60166). Micakwartsbreccie, top Goh Lemboeh. Bruin gesteente. O.h.m.: kwartskorrels van varieerende grootte en vorm, kwartsietbrokjes en resten van kleischalie, door een sericitisch bindmiddel aaneengekit.

No. 21 (60172). Kwartsiet, even beneden triang. pilaar Goh Lemboeh. Grauwgrijs gesteente met kwartsaders. O.h.m.: granoblastisch maaksel van niet of weinig unduleus uitdoovende kwarts. Om de kwartskorrels een donkere, isotrope substantie. Sporadisch muscovietschubjes, toermalijnzuiltjes en een biotietblaadje.

No. 22 (60173). Kleisteen, topzone Goh Lemboe. Brokkelend, bruin, fijngelaagd gesteente. O.h.m.: sericitisch vilt met zeer kleine limonietkorreltjes.

No. 23 (60174). Kwartsiet, westtop G. Kemiri. Wit gesteente. O.h.m.: unduleus uitdoovende kwartsfragmenten, waarbij de kleinere korrels tusschen de grovere liggen, zoodat het geheel een mortelstructuur vertoont (zie ook lit. 2, p. 87).

No. 24. Zandsteen, G. Kemiri. Wit en groen gevlekt gesteente. O.h.m. unduleuse tot kataklastische kwartskorrels, korrels fijnkristallijne hoornsteen, sericitische kleizandsteen, muscoviet en sericiethoopjes. Accessorisch zirkoon en toermalijn. Bindmiddel zeer ondergeschikt: limoniet. Het gesteente bestaat uit detritus van kristallijne schisten en epimetamorfe sedimenten (hoornsteen en kleizandsteen).

No. 27. Travertijn, koepels der heetwaterbron Ajer Panas in de kali Merpoenga. Geelwitte, caverneus schalige $CaCO_3$ afzetting met zwartachtigen aanslag van organischen aard (?algen).

Behalve de bovenbeschreven gesteenten werden een klein aantal zandmonsters, veelal meerdere lagen van één profiel omvattend, medegebracht. Van deze monsters werden de zware fracties onderzocht op de in de sediment-petrografie gebruikelijke wijze. (Zie o.a. C. H. EDELMAN, De Petrologische provincies in het Nederlandsche kwartair). Het resultaat hiervan is neergelegd in onderstaande tabel.

[1]) De nummers 12, 13 en 24 werden naar het Hoofdbureau-Mijnbouw opgezonden en door Dr. VAN BEMMELEN gedetermineerd.

Verzamelnummer	No. Bodemk. Inst.	Vindplaats	Diepte bemonstering in cM.-oppervl.	Opaak	Doorzichtige korrels in onderlinge procentische verhouding							
					Toermalijn	Zirkoon	Rutiel	Epidoot	Amfibool	Augiet	Hypersteen	Monaziet
3	60158	Losir Bivak 3	0—1	35	30	57	9	3	—	1	—	—
4	60159	idem	40	30	20	72	8	—	—	—	—	—
5	60160	Losir Bivak 3	5—10	45	11	71	9	—	7	—	2	—
6	60161	idem	30—35	41	10	44	—	—	41	—	5	—
7	60162	idem	60—65	30	15	58	9	—	13	3	2	—
16	60167	Top Loh Lemboeh	0—10	68	10	71	11	—	1	4	—	3
17	60168	Top Loh Lemboeh	0—5	78	24	74	—	—	1	1	—	—
18	60169	idem	10	82	4	30	—	—	4	60	2	—
19	60170	idem	20	46	13	87	—	—	—	—	—	—

1. Een type, waarin geen vulkanische invloed is te herkennen, resp. deze zeer ondergeschikt is, en waarin slechts toermalijn, zirkoon en rutiel belangrijk zijn (60158/9), terwijl in één monster monaziet werd aangetroffen (60167).

2. Een type met sterk vulkanischen invloed, waarbij amfibool aan de zware mineralen werd toegevoegd (No. 60160/60162), en rutiel afneemt.

3. Een type, waarin augiet de vulkanische bijmenging duidelijk maakt en rutiel geheel ontbreekt (No. 60169).

Opvallend is, dat de bijmenging met vulkanisch materiaal niet in alle onderzochte lagen optreedt, doch zich in het bijzonder bij het profiel van Ngo Lemboeh beperkt tot de middelste laag. Van de overige monsters onderscheidt dit profiel zich bovendien door het ontbreken van rutiel.

SUMMARY.

Petrography and Mineralogy of some rocks and sand-samples from the Gajo Loeeus (Atchin).

Description of sandstone, quartzite, shale, mica-quartz-breccia, claystone, feldsparbasalt and pyroxene-amphibole-andesite from this, hitherto totally unknown district of N. Sumatra. The andesite and one quartzite present indications of contactmetamorphosis and hydrothermal metamorphosis, indicating the presence of acid intrusions in the neighbourhood.

The sands are clearly derived from old, clastic rocks, partly infected with volcanic material.

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Biochemistry. — *Behaviour of microscopic bodies consisting of biocolloid systems and suspended in an aqueous medium.* III. *Coacervation phenomena in droplets of biocolloid sols enclosed in a collodion film. Accumulation of basic dyes.* By H. G. BUNGENBERG DE JONG and O. BANK. (Communicated by Prof. J. VAN DER HOEVE.)

(Communicated at the meeting of December 17, 1938.)

1. *Preparation of the collodion films mentioned in the title.*

In the following a simple preparation of the films mentioned in the title is described in broad outlines, while other methods of preparation and questions concerning details will later be discussed more elaborately elsewhere.

The principle of the preparation is this that in a collodion solution, which cannot be mixed with water, the aqueous solution of biocolloids is emulsified, this emulsion in a thin layer is spread on an object glass and, after gelatination of the collodion, the organic solvents are washed out with water. In order to fix the collodion film to the object glass, the latter after heating is rubbed with a piece of bee-wax on one end and in the middle. After cooling the emulsion, which will be indicated later on, is poured over the surface, the object glass is for a while held in vertical position to enable the excess to trickle down, then turned with the wet side downwards and left to dry in the air fastened in horizontal position in a stand-clamp during a sufficiently long time (e.g. 20—30 min.).

The emulsion to be poured on consists of one volume of aqueous solution of one or more biocolloids in 6 volumes of the emulsifying medium. This emulsifying medium is prepared by adding 45 cc of isoamyl alcohol and 45 cc of ether to 1 vol. 10 % collodion solution (für technische u. fotografische Zwecke RIEDEL–DE HAËN).

After sufficient drying the object glass is placed in distilled water in order to remove the amyl alcohol and any salts which may be present. If the collodion membrane has not become sufficiently strong, it is possible that the thin walls of many, if not of all cavities in the membrane burst and the contents are washed away. Even in the best preparations there will always be cavities in which none of the coacervation phenomena which will be described afterwards take place, since they have been emptied by a lesion. The methods for the correction of these difficulties we shall revert later.

The microscopical image of the collodion film, prepared and washed after the method which has been described, shows that the cavities are

usually surrounded by a condensed zone of the collodion, which gives the impression of a "membrane".

2. *Complex coacervation of a sol mixture of gelatin and gum arabic enclosed in the cavities.*

a. Choice of the biocolloid combination.

It was our object to study the morphological phenomena occurring in case of coacervation of biocolloids present in the cavities. In the first place was selected the so-called complex coacervation, i.e. the mutual flocculation of two oppositely charged biocolloids, the flocculated substance having the nature of a liquid ("coacervate"). Here the conditions which make coacervation possible are the most simple from a biological point of view.

It is only necessary that the P_H reaches a value where the one colloid component (e.g. clupeine, gelatin, serum albumin, egg albumin) has a sufficiently positive charge, the other colloid component (e.g. gum arabic, nucleinate, chondroitin sulphate) still a sufficiently negative charge and besides the two colloid components must be present in a fairly favourable mixing-ratio.

For our purposes egg albumin and serum albumin cannot be used, since denaturation sets in upon contact with the emulsifying medium containing amylalcohol-ether. In the following we restrict ourselves to the combination gelatin-gum arabic, in which case complex coacervation is very easily realized. However, for the microscopical examination it has the drawback that it requires a temperature which is at least 30° C. or higher. It is true, at room-temperature, shortly after cooling of the gelatin sol, still liquid coacervates are formed, but these gelatinize soon, accompanied by simultaneous vacuolization phenomena. In order to be able to microscopize at room-temperature, a preliminary treatment of the gelatin is necessary.

b. Preparing of gelatin preparations with a reduced gelatinizing capacity.

It is possible by means of a suitable preliminary treatment of the gelatin with acids or bases at higher temperatures to weaken the gelatinizing capacity to a sufficient extent, so that preparations are obtained which at room-temperature may be used for our purpose. A too long or too powerful treatment with the mentioned substances should be avoided, since the gelatin ultimately is so strongly affected that complex coacervation with gum arabic is no longer possible.

It stands to reason that directions of general validity cannot be given, owing to the very different characteristics of the various kinds of gelatin on the market.

The following directions, applicable to "Gelatin for bacteriological purposes" of the Glue and Gelatin Works "Delft" at Delft, may give an impression of the formation of preparations obtained by acid and alkaline reactions respectively.

A. (*obtained by acid reaction*).

20 gr. of gelatin are left in 100 cc of distilled water at room-temperature to swell during a sufficient time, then the gelatin is dissolved at a higher temperature, the solution is boiled in a boiler with a round bottom and back-flow cooler, 5 cc of glacial acetic acid being added. After boiling during 1 hour the substance is cooled, 10.8 gr. of Na-acetate being added, left for some time and then dropped into 500 cc of alcohol. The separated tough gelatin mass is now repeatedly treated with 200 cc of alcohol. The removal of Na-acetate and acetic acid is promoted by picking the gelatin mass after sufficient hardening into smaller bits. Finally it has to be treated several times with acetone and dried in the air. Yield — 14.5 gr.

The obtained preparation has not entirely lost the capacity of gelatination and does not dissolve completely in cold water but needs some heating. The 3 % solution at room-temperature remains liquid for a long time, but yet it gelatinizes in the end.

B. (*obtained by alkaline reaction*).

20 gr. of gelatin are left to swell in 180 cc distilled water during 1 hour, after which time the flask is placed during half an hour in a thermostat of 50°, in order to dissolve the gelatin completely. Now 4 gr. of NaOH dissolved in 6 cc of distilled water are added. Since alkali in this concentration and at this temperature (50°) affects the gelatin much more quickly than acetic acid in the abovementioned method, only a short time of reaction should be allowed. At special intervals 50 cc are pipetted from the flask into small flasks with 3 cc of glacial acetic acid. This amount of glacial acetic acid is chosen in such a way that after neutralization of the NaOH a buffer mixture Na-acetate-acetic acid is formed in the ratio of roughly 1 : 1.

It becomes now apparent that by a treatment of 10 minutes a suitable gelatin solution is obtained, which at room-temperature within 24 hours does not gelatinize and still produces complex coacervation. After a reaction of 20 minutes, however, the gelatin has broken down too far.

As was indicated above under *A*, we can of course separate the gelatin again from this solution; however, this is not quite necessary for the following experiments, since the salts impeding coacervation may later be diffused.

c. *Morphology of the complex coacervation of the sol mixture enclosed in the cavities.*

A 5 % solution of the gelatin preparation *A* is prepared or the gelatin solution obtained according to *B* is diluted to half its concentration and mixed with an equal volume 5 % solution of gum arabic [1]).

According to the method indicated in 1. now a collodion film is prepared, in which drops of this sol mixture are enclosed. If the object glass is laid in a PETRI dish and this is filled with glacial acetic acid 250 × diluted with distilled water and placed on the object table of the microscope, we can observe that the complex coacervation now taking place is formed in the following way (cf. also fig. 1):

a. Gradually more and more coacervate droplets are formed in the cavities;

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[1]) We used a very good quality in clear pieces (Gomme Senegal, petite boule blanche from ALLAND et ROBERT, Paris) which after being crushed was dissolved in water. It is to be recommended to centrifuge this solution.

β. They increase in number and size to such an extent as to touch each other and fuse;

γ. The events in *b*. lead to a "phase reversal", i.e. the collodion wall is completely moistened by the coacervate and this fills the whole cavity, a large number of small vacuoles being embedded in it;

δ. The vacuoles are gradually fused, soon one large vacuole is formed by the side of not yet fused smaller vacuoles, and finally the image is found of a coacervate moistening the wall with one central vacuole.

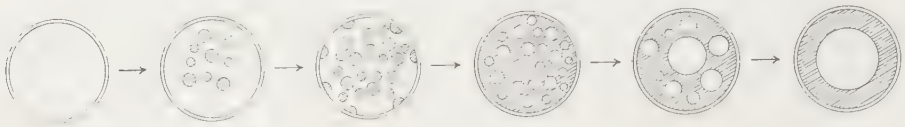

Fig. 1.

On the first microphotograph reproduced here (fig. 2) the last stages of this process of fusion are represented. In the small cavities, where the events succeed each other more rapidly than in the large ones, the final stage is often already reached. In the large ones, on the other hand, a central vacuole is present but the coacervate moistening the wall still contains many smaller vacuoles. On the microphotograph also some cavities are to be seen which contain no complex coacervate. Owing to a lesion, they have already been emptied but consequently they show the more clearly the condensed zone of the collodion round the cavities, which condensed zone makes the impression of a membrane.

Further researches concerning the model described here will be published later in Protoplasma.

Observation.

Although the gelatinizing capacity of the previously treated gelatin has strongly decreased, it has not been completely neutralized. In the complex coacervate this characteristic becomes again clearly manifest. Consequently together with the coacervation gelatination sets in again and only the smaller cavities reach the final stage; the larger ones on the other hand often do not. By heating the dilute acetic acid to 40—50°, here also the final stage may be reached. Accelerated fusion of the vacuoles in large cavities, however, is also possible at room-temperature, if to the dilute acetic acid a sufficient quantity of resorcin is added (e.g. in addition to 0.4 % of acetic acid besides 5 % of resorcin). This fusion may also be brought about by locally in the PETRI dish dropping some drops of concentrated resorcin solution on the preparation. Resorcin, namely, in a sufficiently strong concentration neutralizes the gelatination, but there is a possibility that now in the coacervate new small vacuoles are formed, since resorcin by the side of this property also causes dehydration in a certain range of concentrations.

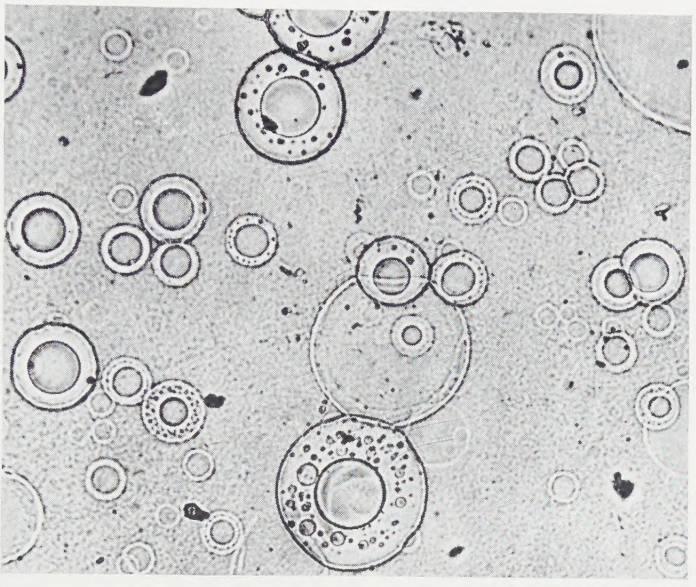

Fig. 2 (180 ×).

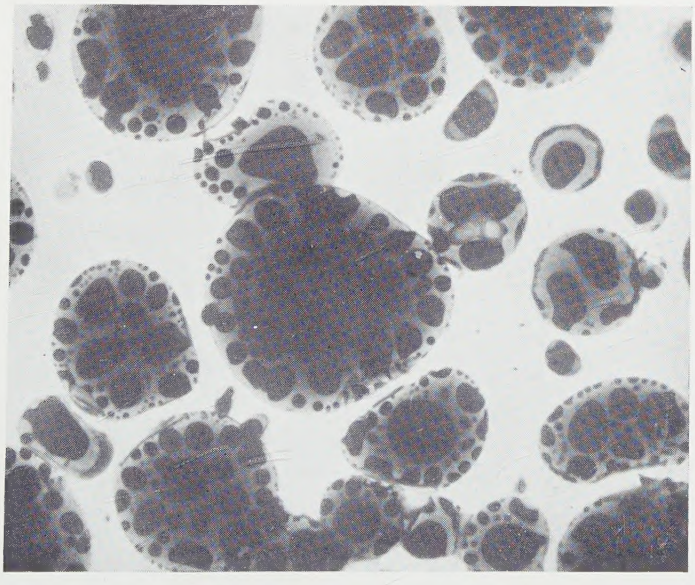

Fig. 3 (200 ×).

d. Significance for biology: Morphological model for parietal cytoplasma with central vacuole.

The adult cell of the higher plants is usually characterized by the possession of one large central vacuole. In young cells this is not yet present, but is gradually formed by fusion of a large number of smaller vacuoles, owing to which phenomenon ultimately the cytoplasma (except plasma strands) comes to be situated entirely against the wall.

Modern cytologists think that the cytoplasma is usually a liquid and that this neither on the outer surface nor on the interface with the vacuole possesses a microscopically visible membrane. In 1932 one of us suggested the possibility that the cytoplasma (and possibly other cell components) might have the nature of a coacervate or at any rate that upon formation of morphological structures coacervation phenomena might be temporarily placed in between [1]). What has been said under *c.* shows that the typical morphological constellation: wall — cytoplasma — vacuole, as well as the formation of the central vacuole by fusion of many smaller ones, may be imitated with models in which coacervates take part.

3. *Accumulation of a basic dye in the cavities containing gum arabic solution and coacervation of the gum arabic in case of sufficient accumulation.*

a. Accumulation of basic dyes.

The following refers to analogously prepared collodion films, with this difference that instead of the sol mixture exclusively a 5 % solution of gum arabic was emulsified in the amylalcoholic-etherial collodion solution.

If a membrane prepared in this way is laid in a dilute solution of basic dyes, e.g. neutral red, methylene blue, brilliant cresyl blue, toluidine blue, fuchsin, etc., accumulation of the dye takes place in the enclosed gum arabic solution.

In principle DONNAN's equilibrium may account for this accumulation. The arabinate ion cannot diffuse, but the cations belonging to it (mainly Ca-ions) will exchange with the dye cations till the ratio of the concentrations (activities) of the anorganic and dye cations in the gum arabic solution is equally large as in the outer liquid. If now the concentration of the original dye solution is low as compared to the concentration of the gum arabic, then after the equilibrium being reached, the dye concentration in the gum arabic solution will be considerably higher than in the outer liquid.

b. Coacervation of the gum arabic.

When as the result of the exchange mentioned in *a.* the substitution of the original anorganic cations, accompanying the arabinate ion, by dye cations exceeds a certain value, coacervation of the gum arabic takes place.

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[1]) H. G. BUNGENBERG DE JONG, Protoplasma, 15, 1932.

In the cavities small deeply stained coacervate drops are separated, which may fuse and are also deposited on the wall. We refer to the microphotograph (fig. 3) where this coacervation has been formed after the preparation being placed in a c. 0.02 % solution of toluidine blue.

c. Significance for biology: Vital staining of vacuoles, unmixing of the vacuolar liquid.

The above-described accumulation of basic dyes and possible following phenomena of unmixing may perhaps be regarded as models for the vital staining of vacuoles in plant cells and for the unmixing phenomena which may accompany them. It has to be assumed then that diffusible cations from the vacuoles exchange with dye cations. However, there is this difference that this model shows the described phenomena with all typical basic dyes, whereas in plant cells only a group, the so-called vacuolar dyes, is able to bring this about. This may be due to differences between the dyes mutually as regards the penetrability of the protoplasm: The vacuolar dyes can pass this as a whole, the others cannot.

Further particulars concerning accumulation of basic dyes and the coacervation following a very strong accumulation, as well as a discussion as to how far the mechanism of vital staining of the vacuole might be explained in this way, will be published later in Protoplasma.

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